Market Power and Price Informativeness*

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Abstract

We study the distributional effects of asset ownership on price informativeness in a general equilibrium model featuring investors (oligopolists) with different degrees of price impact and ability to learn about individual asset payoffs from private signals as well as price signals, and competitive fringe that only learns from asset prices. We show that price informativeness is non-monotonic in the oligopolists' aggregate size, decreasing in the sector's concentration and in the size of the passive oligopolistic sector. We further show that the size effect can be decomposed into a learning channel capturing investors' quality of private signals and an information pass-through channel measuring the sensitivity of investors' trades to private signals, with the latter one being the primary source of variation in price informativeness relative to the size distribution.

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1 Introduction

Investing in financial assets is one of the major cornerstones of wealth accumulation. The demand side of financial markets is typically divided between institutional investors, often distinguished by their large size and the amount of information they produce[1], and retail investors, who are small and relatively less informed. The distribution of asset ownership and its impact on market stability and welfare has attracted considerable attention from market participants, policy makers, and academics. The proponents of a growing asset management business point out that the larger sector can accommodate the growing demands for saving vehicles, including endowments, pensions, or individual savings, at a much lower cost. The adversaries argue that excessive market size converts the active portfolio management business, which induces faster price discovery and greater price informativeness (alpha investing) into a business where investors get exposure to more passive risk premia but at less efficient prices (factor investing).[2] Further, some critics argue that the growing size of the asset management may also imply an increasing concentration of assets in the hands of just a few holders, thus making them systemically important institutions. These financial stability considerations have, for example, motivated the regulation of the largest asset management companies similar to the banking risk management framework.[3]

In this paper, we analyze theoretically the impact of the size, concentration, and active/passive ownership share of large investors on price informativeness.[4] At the heart of our analysis lies an endogenous trade-off between the information acquisition and trading decisions of investors of different sizes. On the one hand, some large active investors may decide to acquire private signals, the use of which could increase the amount of information revealed in asset prices. On the other

[1] In 2019, the institutional ownership of an average stock in the US equaled around 60%. Within the institutional sector, active investors who rely on private information hold about the same share of the market as do passive investors who do not trade for information reasons. The ownership structure is heavily skewed, with the ten largest investors holding, on average, 35% of total shares outstanding, and it varies greatly across individual assets. On a broad basis, the PWC consulting firm projects that the size of global asset management sector will almost double by 2025 reaching a staggering $140 trillion.

[2] For the discussion of merits and downsides of factor investing, see Ang (2014).

[3] For summary of some of these points, see the 2015 Milken Institute Symposium: "How Asset Management Is Reshaping the Global Financial System".

[4] The literature on price informativeness and market efficiency is quite vast and its proper summary is beyond the scope of the paper. The specific measure of price informativeness we use, defined as the covariance of the price with the fundamental, normalized by the volatility of the price, has a strong economic appeal in that it can be derived as a welfare measure using Q-theory. It increases with the correlation between the price and the fundamental, and the volatility of the fundamental, as correlation is more meaningful when the unobserved variable is more volatile. Analytically, it represents the reduction in the variance of posterior beliefs when agents use price as a signal about fundamentals. The above measure has gained significant interest among academics over the last few years, starting with the work of Bai, Philippon, and Savov (2016) and later Farboodi, Matray, Veldkamp, and Venkateswaran (2021). For a broader discussion of theoretical measures of price informativeness, see Davila and Parlatore (2020).

hand, all large investors also recognize their price impact, which makes them trade less on any information they acquire *and* affects their learning choices.

We show that this tradeoff can be captured by two channels that determine the behavior of price informativeness: the *information pass-through channel*, which quantifies the sensitivity of trading decisions to information, and the *learning channel*, which isolates the pure effect of the information choices on portfolios. The interaction of these two channels results in price informativeness that has a non-monotonic relationship with the aggregate share of large investors and a strong negative relationship with the concentration of their ownership. We also show how these channels generate a negative general equilibrium amplification effect on price informativeness from an increase in the size of the passive investing sector through re-optimization of active learning decisions.

To explore the endogenous interaction between learning and trading, we build a new theory that features large investors with varying sizes. Specifically, our baseline setting, which we further extend in various robustness tests, includes a mass of atomless competitive traders, called the fringe, each of whom takes prices as given, and l *oligopolists* of different sizes–investors who know that their trades move prices.[5] Some oligopolists are endowed with a capacity to collect information, which they can use to reduce uncertainty about asset payoffs. They are also able to learn from market prices. Other oligopolists are passive in that they do not collect asset-specific information, and do not learn from the price, but they do internalize the price impact of their demand.[6] Fringe investors do not have any information capacity but they can learn from the public signal of the price. Finally, all oligopolists are strategic in that they respond optimally to other investors' endogenous learning and portfolio decisions across multiple assets, defined to be heterogeneous in terms of their supply process and fundamental volatility.

We model individual learning choices using the theory of rational inattention of Sims (2003). Investors allocate their learning capacity optimally across assets, depending on the assets' characteristics and the investors' objective functions. After learning choices have been made, trading takes place via demand schedule competition. Broadly, our theory extends the work of Kyle (1989) and Vives (2011) to allow for endogenous information choices under non-symmetric allocations of information and trades. The equilibrium is a fixed point involving not only demand schedules but

[5]This modeling assumption stands in contrast to that in the literature with oligopolistic traders and noise traders, in which oligopolists make up 100% of the market; hence, comparative statics with respect to the sector size are trivially ruled out. Additionally, the presence of competitive fringe ensures existence of equilibrium for small number of strategic traders, in contrast to Kyle (1989).

[6]Learning from prices by passive investors does not change any of the qualitative conclusions from the model. The results of this alternative model are available from the authors upon request. Our results are also robust to modeling passive investors as 'pure indexers', as we show in Appendix B.5.

also learning choices across multiple heterogeneous assets and multiple oligopolists of heterogeneous sizes. Both information and trading choices are asymmetric if oligopolists differ in terms of their sizes. The asymmetries allow us to address new questions concerning the impact of large investors' aggregate size and concentration on price informativeness. They have significant impact on the response to growth of investors with differential information capacity; in particular, the passive investment sector.

We derive a set of results on the relationship between the size distribution and price informativeness. In our model, due to asymmetric sizes, the solution implies asymmetric learning strategies among oligopolists. Allowing for such modeling generality requires numerical solutions of the model. We start by analyzing a special case of a single large monopolist, which allows us to focus on pure size effects while abstracting from strategic interactions between oligopolists. We show that the average price informativeness is *non-monotonic* in the size of a monopolist, first increasing, but eventually decreasing to zero as his size grows. The primary force responsible for this non-monotonic behavior is the information pass-through channel. On the one hand, as the price impact of his trades grows, the monopolist reduces trading sensitivity to his signals, revealing less information in prices. On the other hand, size itself increases the importance of his trades for market clearing, meaning they reveal more information in prices. Eventually, in the limits in which size approaches either zero or the size of the entire market, price informativeness goes to zero. In sum, the presence of market power can have a detrimental effect on price informativeness. This result stands in stark contrast to a similar experiment with perfectly competitive atomless investors, in which price informativeness is strictly increasing in investor size.[7]

Next, we analyze the model with a nontrivial ownership structure and study additional questions, such as the effects of concentration and shifts in passive ownership on price informativeness. Our setting considers 20 investors with market power. Among those investors, we allow for some to be larger than others, and we allow for some to be active and others to be passive. This setting allows us to conduct four main sets of experiments.

In the first experiment, we study the model results as a function of size of the oligopolistic sector. We find a hump-shaped response of price informativeness, a result that is qualitatively consistent with the one based on the monopolistic framework. A decomposition of price informativeness into the learning channel and the information pass-through channel reveals that the information pass-through channel is the primary force driving price informativeness. The intuition

[7] We discuss these results in Appendix A.1.

for the result is similar to the monopolistic framework, with an additional quantitative effect of the strategic substitutability in learning across oligopolists. Specifically, we show that a larger size of the oligopolistic sector makes this substitutability stronger and prevents oligopolists from fully diversifying their learning, which ultimately hurts price informativeness.

In the second experiment, we increase the size of the passive sector at the expense of the active sector. This experiment allows us to separate the effect of price impact of large investors from that due to their informational advantage. We show that the average price informativeness generally decreases with the size of the passive sector, with any nonmonotonicity driven by information pass-through. A more nuanced picture emerges for individual assets. As the size of the passive sector increases, the now smaller active investors focus their learning on fewer assets. Specifically, assets in large supply (with higher returns to learning) exhibit a flat or hump-shaped price informativeness, while assets in small supply (with lower returns to learning) observe a decrease in their price informativeness. This heterogenous cross-sectional response to a growing passive sector is observationally consistent with the pattern of cross-sectional changes in price informativeness by asset size, documented in Bai, Philippon, and Savov (2016) and Farboodi et al. (2020). Our results suggest that the observed empirical heterogeneity in price informativeness may be partially due to a rise in size of passive sector.

In the third experiment, we vary the concentration of the oligopolistic sector, holding its total size fixed. We show that an increase in concentration reduces price informativeness, the result driven by both the learning and the information pass-through channels. Intuitively, increased concentration means polarization of sizes and a smaller information pass-through for both the oligopolists that grow in size (because of the growing price impact) and the ones that diminish in size (because of a lower economic importance). At the same time, since investors endogenously adjust their learning, price informativeness is impacted through two opposing forces: the growing oligopolist diversifying learning (positive impact) and the shrinking oligopolist specializing (negative impact). On net, the two channels contribute to the negative relationship between concentration and price informativeness.

Finally, we combine the size and passive share growth experiments, motivated by the trends of a simultaneous growth both in overall sector size as well as in its passive share. We show that in this combined experiment, price informativeness is also hump shaped, driven this time by information pass-through and a growing passive share. In the cross-section of assets, we observe a clear level effect of assets with large institutional ownership exhibiting higher price informativeness. However, in terms of dynamics, we observe that the combination of size and passive share changes gives a relatively stable price informativeness asset by asset. We argue that both of these results are consistent with empirical findings in Bai, Philippon, and Savov (2016). Hence, the conclusion from our analysis vis a vis the data is that passive share is a crucial additional factor that determines the relationship between the size of institutional ownership and price informativeness.

The conclusions from our four main experiments extend to a setting in which we allow oligopolists to choose their information capacity optimally, subject to a convex adjustment cost. A priori, the size distribution could affect incentives to invest in information processing capacity, as price impact diminishes the rents from informed trading. However, we show that for a variety of the cost parameterizations, the optimal capacity choice is relatively stable across market structures, and hence the conclusions from our baseline model with fixed capacity remain similar. Intuitively, as the size of the oligopoly sector varies, two opposing forces shape optimal choice of information capacity. On the one hand, larger size means that the information capacity is applied to a larger size of the portfolio, implying economies of scale and an increase in optimal capacity. On the other hand, larger size is associated with a larger price impact, and hence the rents from better information cannot be fully captured. The interaction of these opposing incentives implies that the variation in optimal capacity is relatively small as the distribution of size changes.

In an additional experiment, we examine the role of endogenous learning. We compare the predictions of our benchmark model with those of a model in which learning choices are exogenously fixed, holding the informational capacity constraint the same across models. We show that fixing information choices significantly affects the conclusions of our experiments. In the experiments that change total size or the size of passive sector, an exogenously fixed information choice determines the size of the oligopolistic sector which maximizes price informativeness. This optimum can be higher or lower than that implied by the endogenous-information benchmark model, depending on the distribution of the exogenous information. For the concentration experiment, we show that fixing information choices can actually overturn the result that price informativeness is decreasing in concentration, giving instead a hump-shaped response for certain parameter values. Intuitively, the only channel that operates in the exogenous information case is the information pass-through channel; hence, the shape of the price informativeness response crucially depends on the information endowment of the oligopolist whose information pass-through changes the most (through his size). We conclude that allowing for endogenous learning is essential when studying the interaction between ownership structure and information content of asset prices.

Apart from our main numerical exercises, we provide an extensive set of robustness results as well as some analytical results in a simplified version of the model. First, in order to gain further insights into the workings and interactions of the two main channels determining price informativeness in our model, we analytically characterize the relationship between price informativeness and size in two polar opposites of our model: competitive and a simplified monopolistic setup. We show that under the competitive assumption, price informativeness is monotonic in size and that under monopoly, it has a humped shape driven by information pass-through. These results are independent of parameter values and give intuition behind the robustness of our numerical results. Second, we consider a simplified duopoly setup where we analyze the optimality of different size distribution for price informativeness, as well as characterize the effects of one duopolist information and size on the other's information pass-through. Third, we provide numerical results from our three experiments for alternative specifications of the model that assume decreasing absolute risk aversion specification, alternative linear specification of the entropy constraint, modeling passive investors as pure indexers or allowing for the existence of positive capacity retail investors. In Appendix B, we show that these alternative specifications do not alter the qualitative conclusions from our analysis.

1.1 Related Literature

Our paper straddles two types of literature: the empirical one on price informativeness and trading that motivates our broad investigations in this paper, and the theoretical one that helps us to build the foundation of our work. Specifically, on the applied side, our paper connects to the growing literature on price informativeness. Bai, Philippon, and Savov (2016) show that price informativeness is greater for stocks with greater institutional ownership. Our model delivers such a result for a range of ownership values. However, our theoretical analysis implies that, beyond certain levels, ownership may in fact reduce price informativeness, due to excessive price impact. Our micro-founded equilibrium model allows us to study the underlying economic mechanism in depth, as well as additionally investigate the role of ownership concentration and passive ownership. Bai, Philippon, and Savov (2016) and Farboodi et al. (2020) examine differences in price informativeness between companies included and not included in the S&P 500 index, as well as stocks sorted by institutional ownership. They show that the indexed companies exhibit larger efficiency and are the only ones to exhibit an increase in price informativeness, which they attribute to a composition effect of these companies, being older and larger. They also find higher price informativeness for high institutional ownership stocks, with the difference persisting but not growing over time. We show that the predictions of our model are consistent with these empirical findings, suggesting that they may be partially due to a rise of passive investing, and in general are consistent with a simultaneous rise in institutional ownership and passive investing. Kacperczyk, Sundaresan, and Wang (2021) show that the stock ownership by active institutional investors, domestic and foreign, causally increases price informativeness of stocks in which they invest more.

Finally, we add to a growing empirical literature that studies the impact of market structure in asset management on various economic outcomes. Following the diseconomies of scale argument of Chen, Hong, Huang, and Kubik (2004), Pástor, Stambaugh, and Taylor (2015) show significant diseconomies of scale at the industry level. Using a merger between BlackRock and BGI as a shock to market power, Massa, Schumacher, and Yan (2020) study the asset allocation responses of their competitors. Our work complements these studies by providing and quantifying a mechanism through which ownership structure endogenously determines price informativeness.

The theoretical literature on informed trading with market power dates back to Kyle (1985) and Grinblatt and Ross (1985) whose setups feature one strategic trader, and Kyle (1989) and Holden and Subrahmanyam (1992), who extend the model to an oligopolistic framework.[8] The effects of market size on price informativeness and efficiency have been studied in models of oligopolistic financial markets by Vives (2011), Rostek and Weretka (2012), and Vives (2014). Lambert, Ostrovsky, and Panov (2018) further extend the Kyle (1989) model to study the relation between the number of strategic traders and information content of prices in a general stochastic environment. Kyle, Ou-Yang, and Wei (2011) allow for endogenous information acquisition in a one-asset economy, but their mechanism focuses on the contracting features of delegation. Finally, Xiong and Yang (2021) examine the role of price feedback effects in the oligopolistic market for firm disclosure and price informativeness.

Apart from the main difference that none of the above theoretical papers study the questions which relate to the distributional properties of asset ownership, our work also differs from this part of the theoretical literature in several modeling aspects. The first important innovation is that all of the papers assume either a single large trader, or a set of symmetric (in size) large traders. Our model allows for arbitrary distributions of size and market power, which is necessary for us to be able to characterize the impact of sector concentration. In this respect, our paper is the first

[8] Models in which traders condition on others' decisions also include Foster and Viswanathan (1996) and Back, Cao, and Willard (2000).

information-driven treatment of the effects of sector concentration on price informativeness. The second important innovation is that our framework features multiple, heterogeneous assets under endogenous information acquisition, which implies clear preferences of investors in terms of learning choices. This allows the model to have testable predictions in terms of cross-section allocation of holdings in response to the growth of the passive investment sector. The third innovation is how we model size, or market power. The literature typically uses either a market maker (or market making sector) or atomistic traders. As a result, the relative 'size' or 'market power' of informed participants only gets adjusted if the total number of informed or uninformed traders changes. Our framework adjusts size and market power of individual traders by changing their assets under management, making it relevant for our motivating empirical evidence.

Our general equilibrium model builds on the literature on endogenous information choice, in the spirit of Sims (1998, 2003). More closely related to our application are the models of costly information of Van Nieuwerburgh and Veldkamp (2009, 2010), Mondria (2010), Kacperczyk, Van Nieuwerburgh, and Veldkamp (2016), and Kacperczyk, Nosal, and Stevens (2019). Ours is the first study to introduce (heterogeneous) market power into a model with endogenous information acquisition. This aspect allows us to study strategic responses of oligopolistic traders in terms of their demand and information choices. Within the theory of rational inattention, we show that the framework with large investors leads to an interior learning solution, a contrast to the studies with competitive agents, in which each investor learns about one asset only. Finally, and very distinctly, the competitive framework under rational inattention would imply a monotonic relation between aggregate size and price informativeness, whereas this relation becomes hump shaped when price impact is explicitly modeled.

More generally, our study can be placed in the literature on information production and asset prices. Bond, Edmans, and Goldstein (2012) survey the literature on information production in financial markets, emphasizing the differences between new information produced in markets (revelatory price efficiency: RPE) and what is already known and merely reflected in prices (forecasting price efficiency: FPE). Our focus is solely on RPE, which is largely dictated by the modeling framework we use, and is consistent with the focus on RPE in Bai, Philippon, and Savov (2016).[9] Goldstein and Yang (2015) study incentives for trading on private information and their effect on price informativeness, identifying effects that can be mapped to our measure of information

[9]Theoretical work on asset prices and real efficiency also includes Dow and Gorton (1997), Subrahmanyam and Titman (1999), Kurlat and Veldkamp (2015), and Edmans, Goldstein, and Jiang (2015).

pass-through in a setup with exogenous information. We show that modeling endogenous information acquisition is crucial in drawing conclusions about price informativeness, as the information structure effectively determines the shape of the price informativeness response to different market structures. In a broader institutional context, Stein (2009) develops a model of market efficiency and sophisticated (arbitrage) capital in the presence of capital constraints. Garleanu and Pedersen (2018) examine the role of search frictions in asset management for price efficiency. Breugem and Buss (2019) study the impact of benchmarking on price informativeness in a costly information acquisition competitive equilibrium model.

A growing literature has also started to study the impact of passive investors on asset prices, and that of large index investors on price efficiency. Bond and Garcia (2022) show that increased indexing due to, for example, a reduction in indexing costs reduces price efficiency of the index but can augment relative price efficiency. Stambaugh (2014) argues that reduced retail trading allows for less noisy prices which leads to increased price efficiency. Sammon (2021) proposes three newer definitions of price informativeness and argues empirically that passive investing has been tied to reductions in those measures. Buss and Sundaresan (2020) show that increased passive investing can lead to an increased investment by firms, which in turn can cause active investors to pay more attention, raising price efficiency. Our paper adds to this discussion by analyzing the impact of passive investing in a world where large institutions have price impact, and shows that aggregate price efficiency declines.

The rest of the paper is organized as follows. In Section 2, we present a set of motivating facts on institutional ownership and its concentration. Section 3 presents the theoretical framework and the equilibrium concept. In Section 4, we derive numerical solutions for the baseline model and discuss comparative statics. Section 5 concludes.

2 Motivating Facts

In this section, we present a set of stylized facts that motivate our paper. First, we show that institutional investors in many developed economies hold a large fraction of equities. Second, using U.S. market data, we demonstrate a large and increasing role of passive institutional investors in the ownership structure. Finally, the holdings of the largest institutions, a measure of ownership concentration, make up a significant share of total equities in most markets. Our definition of institutional investors means large asset management companies and not individual fund portfolios.

This definition of investor type matters in the context of ownership concentration, which is known to be highly skewed towards large asset management companies. While our figures show time-series trends, our focus is more on the static effects of market structures.[10]

The facts we document are derived from global institutional stock ownership data from Factset. Factset provides comprehensive information on institutional ownership of equity from over 40 countries. These data are considered the most comprehensive in the market and cover more than 98% of total value of publicly listed companies worldwide. The data are measured at a quarterly frequency and span the period 2000–2017.

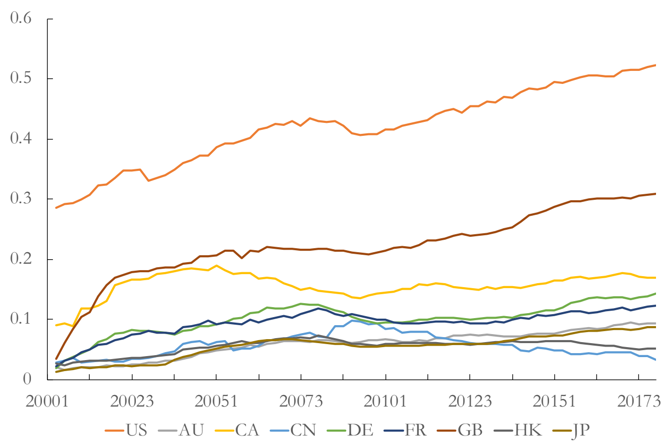


Figure 1: Institutional ownership worldwide.

Our first quantity of interest is institutional ownership, calculated as the stock-level share of stocks held by financial institutions at the end of a given year relative to the number of shares outstanding. Since we are interested in market-level quantities, we take simple averages across all stocks in our sample. Because institutions tend to favor large companies in their portfolios we use equal weighting, rather than value weighting, as a more conservative metric of the patterns in the data. We present the data for the largest equity markets worldwide, including Australia, Canada, China, France, Germany, Hong Kong, Japan, the United Kingdom, and the United States. We present the time series of institutional ownership in Figure 1. We observe a large cross-country variation in the importance of institutional owners. Market-based economies, such as the U.K. and the U.S., have the highest levels of institutional ownership, with the U.S. having an average ownership of almost 60%.[11] Bank-based economies, such as Germany, France, and Japan, have lower

[10] A parallel microstructure literature (Boehmer and Kelley (2009)) examines empirically the relation between institutional ownership and price efficiency due to trading intensity. Efficiency there is measured using variance ratios and pricing errors. Their conclusions are akin to ours.

[11] This number reaches almost 80% when we aggregate ownership using market weights of individual firms.

levels of institutional holdings. However, except for China, all of these markets have witnessed a rapid increase in institutional ownership by 200% to 300% over the last 20 years. As institutional investors currently represent a significant percentage of total global asset holdings, questions of their optimal size are of increased importance.

Next, we present the distribution of ownership with respect to institutions' information. In our data, active investors are those engaged in information acquisition and passive investors are those who strictly invest in index portfolios and do not invest in information acquisition. The latter group includes both index mutual funds and ETFs. Since identifying passive funds in the global context is difficult, we use the evidence from the Investment Company Institute (ICI) Fact Book. This source restricts our analysis to the U.S. market alone, though we believe that similar patterns are likely in other markets as well. We calculate the share of passive ownership in total stock ownership of institutional investors and present the results in Figure 2 for the period 1993–2017. We observe a significant increase in passive ownership over time. While in mid–1990's passive funds accounted for less than 5% of institutional equity market size in the U.S., this share has increased to 45% by 2017. In fact, as of December 2020, passive funds hold a greater share of equities than active funds.

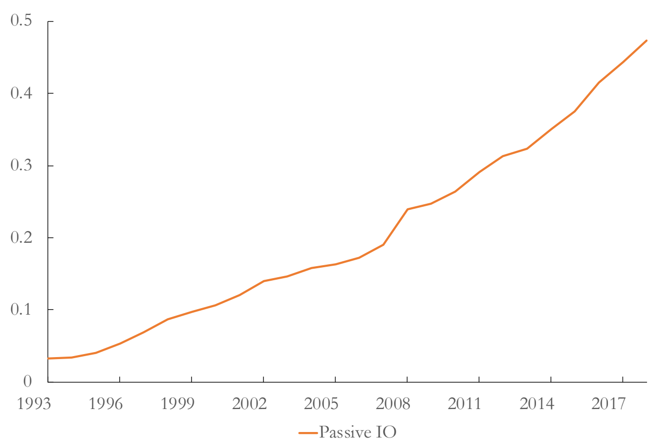


Figure 2: Passive ownership as a share of total institutional ownership.

Finally, we focus on market concentration. We define concentration as the ratio of the holdings of the top-5 largest institutions to total institutional holdings for a given stock. Notably, the definition of an institutional investor in our data is an asset management company, such as Fidelity or Vanguard. In this respect, an institution could reflect a collection of individual funds. For example, Fidelity would comprise Fidelity Magellan Fund, Fidelity Growth Fund, etc. Even though the choice of the investor definition is not crucial for our model and the predictions we derive, using institutional companies as proxies for investors matches more closely the framework in which some

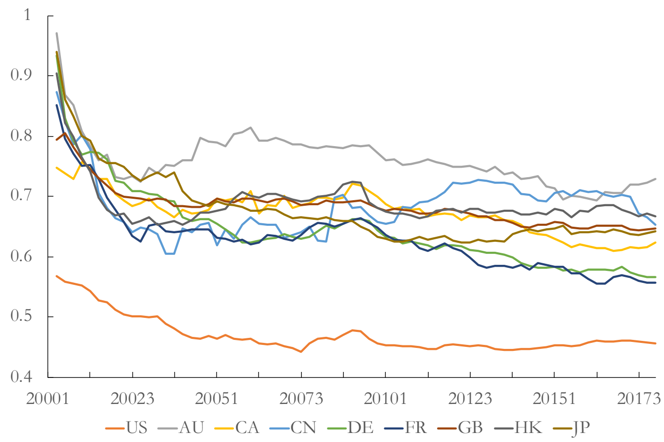


Figure 3: Top-5 investors as a share of total institutional ownership.

investors have market power. Moreover, the patterns we observe in the data indicate a high degree of concentration of stock ownership in the hands of only few institutions represented by asset management companies, such as Blackrock, Fidelity, or Vanguard.[12]

To show the concentration data formally, we average the ratios across all stocks in each period. We present the time-series evolution of the country-specific quantities in Figure 3. Despite their different levels of institutional ownership, all the markets exhibit a high degree of market concentration, between 50% and 80%. In contrast to the steady increase in the size of institutional investors over the past 20 years, the concentration levels have been relatively stable over time. Of course, given the increase in total ownership of the sector, the largest players have increased their presence in the market, which makes their impact potentially much more significant. This fact highlights the growing economic importance of investors with market power.

3 Model

In this section, we present the model of investor trading behavior which aims to speak to the facts of Section 2. Our model features information and portfolio choices of investors who are constrained in their capacity to process information about asset payoffs. We allow for asset heterogeneity in supply and fundamental volatility, as well as for investors to differ in their information capacity and size.[13] Some of the investors are large, meaning that their trades have price impact, which

[12]This evidence contrasts the one documented at the portfolio (fund) level (e.g., Feldman, Saxena, and Xu (2020)), suggesting that the concentration at that level has been going down over time.

[13]In the numerical section, we specifically focus on supply heterogeneity, but our results apply more generally. As a robustness, we have also calculated results from a model with asset heterogeneity in the form of fundamental volatility. Heterogeneity in that factor also provides a clear ranking of the returns from learning about an asset. The

they internalize. In equilibrium, this feature affects both their information and portfolio choices. Since many of the empirical facts motivating our paper are cast in the context of institutional investors, we distinguish theoretically between institutional and retail investors. In fact, it is quite natural to label some investors in our model as institutional investors for at least two reasons. First, such investors have the ability to gather private information, as has been documented in numerous empirical studies. Second, they may exert price impact due to their economic size. Both features are essential components of our modeling framework. Even though our model is not designed to confront all aspects of institutional trading, it offers a good way to capture an economic mechanism driving the patterns we observe in the data.[14]

Setup A unit continuum of investors is divided into two groups. One group, the *competitive fringe*, is of mass $\lambda_0 < 1$ of atomistic uninformed investors, indexed (collectively) by $j = 0$.[15] The second group consists of l-many *oligopolists*, indexed by $j \in \{1, ..., l\}$, each with positive information-gathering capacity K_j and size λ_j, such that $\sum_{j=0}^{l} \lambda_j = 1$. The sizes of the oligopolists, parameterized by λs, map monotonically into ownership; hence, in our experiments, we use them as proxies for ownership shares. Oligopolists are large in the sense that they have positive mass, and hence price impact, which they internalize. We divide oligopolists into four groups: (i) large active investors $j \in LA$, who have large mass λ_j and capacity K_j; (ii) large passive investors $j \in LP$, who have large mass but zero capacity; (ii) small active investors $j \in SA$, who have relatively small (but positive) mass and price impact, and lower capacity than large active investors; and (iv) small passive investors $j \in SP$, who have small (but positive) mass and zero capacity. The optimization problem of each group of investors below differs only in terms of their size λ_j and capacity K_j and hence can be presented in generality. However, for the purposes of mapping our model to the ownership structure data, which is characterized by a relatively high level of concentration of institutional ownership and the presence of both passive and active investors, it is helpful to explicitly distinguish among the four types.

conclusions from the experiments are qualitatively similar.

[14]Notably, our data patterns, especially those on concentration, correspond mostly to asset management companies. Hence, this is the definition of investors we assume in our model. Even though the typical portfolio choice problem is solved at the individual fund level, it is not unrealistic to assume that some large institutional players internalize their price impact at the organizational level. In fact, the literature on the market structure of asset management companies (e.g., Sharpe (1981), Elton and Gruber (2004), and Kacperczyk and Seru (2015)) argues that when it comes to risk management many fund families optimize their decisions at the asset management company level. Similarly, many asset management companies aggregate information across their individual portfolio units.

[15]The presence of fringe investors allows us to naturally construct experiments in which we vary the size of the overall large investor sector share relative to the total. Crucially, in contrast to Kyle (1989), the presence of a competitive fringe ensures the existence of an equilibrium for an arbitrary number of strategic traders.

Each investor solves an information capacity allocation problem and portfolio choice problem to maximize the expectation of mean-variance utility over end-of-period wealth, with a common risk aversion coefficient, $\rho > 0$.[16]

The financial market consists of a risk-free asset, with price normalized to 1 and net payoff r, and $n > 1$ risky assets, indexed by i, with prices p_i and independent payoffs $z_i = \bar{z} + \varepsilon_i$, with $\varepsilon_i \sim N(0, \sigma_i^2)$.[17] The risk-free asset is in unlimited supply, and each risky asset has supply $x_i = \bar{x}_i + \nu_i$, with stochastic shocks $\nu_i \sim N(0, \sigma_{xi}^2)$, independent of payoffs and across assets. The shocks are meant to reflect the demand of non-optimizing noise traders, who trade for reasons orthogonal to prices and payoffs (e.g., liquidity, hedging, or life-cycle reasons).[18]

Investors know the distributions of all shocks, but not their realizations (ε_i, ν_i). Prior to making their portfolio decisions, active investors ($j \in SA \cup LA$) can obtain information about some or all of the risky asset payoffs in the form of private signals. All active oligopolistic investors and the competitive fringe learn from prices as well.[19] The quality of the private signals is constrained by each investor's capacity to process information, $K_j \geq 0$, which places a limit on the reduction of uncertainty about asset payoffs. We model the constraint as a capacity for entropy reduction (Shannon (1948)), following the work of Sims (2003). Higher capacity can be interpreted as having more resources to gather and process news about different assets, and it translates into signals that track the realized payoffs with higher precision. Active investors choose how to allocate attention across different assets. After observing their private signals, active investors and the fringe also observe the price and update their beliefs. Prices adjust endogenously to clear markets. Oligopolists are strategic and directly reason through the market clearing equation in order to determine their own price impact as well as how much to update their beliefs from the price signal. All oligopolists' informational capacities K_j are common knowledge.

[16] The mean-variance utility assumption is standard in the literature and is known to provide tractability to the model, allowing for at least partial analytical characterization of equilibrium. As a result of this assumption, our model does not feature wealth effects.

[17] Under the assumption of independence of signals across assets, independence of payoffs occurs without loss of generality, as asset payoffs can be easily orthogonalized under such assumptions. For a discussion of this issue, see Van Nieuwerburgh and Veldkamp (2010).

[18] In principle, for versions of our model with multiple oligopolists, the introduction of noise traders is not strictly necessary, as the noise in oligopolists' signals will always show up in the price, preventing perfect revelation of fundamentals (Vives (2011) shows that noise traders can be eliminated under certain conditions). By including noise traders, however, we ensure that the models considered in the NRE literature are special cases of this model. We also have asymmetries in learning equilibria, so some assets will only be learned about by one oligopolist. When that happens, without noise traders, the oligopolist's signal is perfectly revealed by the price.

[19] In the benchmark model, we define passive investors as those that do not have the ability collect private information and do not update from the price. The last assumption is about the nature of passive investing in the data. Modifying it to allow for price learning by passive investors does not impact the predictions of the model in a significant way. Results with price learning by passive oligopolists are available upon request from the authors.

Trading strategy We assume (and verify as an equilibrium) that the portfolio strategy of each investor $j \geq 0$ for each asset i takes the form of a linear demand schedule which depends on their private signal about that asset, s_{ji}, and the price p_i, (as in Kyle (1989)):[20]

$$q_{ji} = \beta_{0ji} + \beta_{1ji} s_{ji} - \beta_{2ji} r p_i, \tag{1}$$

where β_{1ji} measures the response of quantity demanded by oligopolist j to his private signals,[21] and the coefficient β_{2ji} measures the responsiveness of the quantity demanded to the price. For passive investors and the uninformed fringe, the signal is just the prior, that is, $s_{ji} = \bar{z}$, but the structure of (1) remains the same.

Given their posterior beliefs, $(\mu_{ji}, \hat{\sigma}^2_{ji})$, each investor chooses a trading strategy, summarized by $\{\beta_{0ji}, \beta_{1ji}, \beta_{2ji}\}_{i=1,\ldots,n}$, as a best response to the other investors' trading strategies $\{\beta_{0ki}, \beta_{1ki}, \beta_{2ki}\}_{k\neq j, i=1,\ldots,n}$, conditional on other oligopolists' learning choices. Hence, for every learning choice by oligopolists, the $\{\beta_{0ji}, \beta_{1ji}, \beta_{2ji}\}_{j=0,\ldots,l, i=1,\ldots,n}$ are a Nash equilibrium.

For the demand schedules submitted by investors to be part of a Nash equilibrium, they must be consistent with utility maximization. Given posterior beliefs of an investor j, $(\mu_{ji}, \hat{\sigma}^2_{ji})$, after observing private signals (for active investors) and the price signal (for active investors and the fringe), utility maximization is:

$$U_j = \max_{\{q_{ji}\}_{i=1}^n} E[W] - \frac{\rho}{2} V[W] \quad s.t. \quad W = (1+r)\bar{W} + \sum_{i=1}^n q_{ji}(z_i - r p_i), \tag{2}$$

where the expectation and variance are conditional on the investor j's information set. Without loss of generality, we normalize initial wealth $\bar{W}$ to zero. The solution to the above problem depends on whether the investor is an oligopolist ($j > 0$) or a member of the competitive fringe ($j = 0$). In particular, the demand *of an oligopolist* j for asset i depends on the degree of impact oligopolist j has on the price of asset i, captured by dp_i/dq_{ji}. Price impact introduces a wedge into the otherwise standard CARA demand function:

$$q_{ji} = \frac{\mu_{ji} - r p_i}{\rho \hat{\sigma}^2_{ji} + r \frac{dp_i}{dq_{ji}}}. \tag{3}$$

[20] This specification is in line with the contributions of Vives (2011), Vives (2014), and Kyle (1989). Here, as opposed to those papers, we must allow for heterogeneous βs, because the oligopolists have endogenously heterogeneous information sets. This flexible setup encompasses demand schedule competition if oligopolist j takes other oligopolists' βs as given and also Cournot competition if oligopolist j takes other oligopolists' qs as given. The Cournot case is equivalent to setting $\beta_{2ki} = 0$ in equation (4).

[21] As we show below, in our setup, the posterior mean is equal to the signal s_{ji}.

Oligopolists internalize their price impact when making their trading decisions, given (1), which, together with market clearing, implies:

$$\frac{dp_i}{dq_{ji}} = \frac{\lambda_j}{r\sum_{k\neq j}\lambda_k\beta_{2ki}}. \tag{4}$$

The impact that oligopolist j has on the market price depends positively on his size λ_j, and negatively on the sizes of the other investors, as well as on the responsiveness of other investors' quantities to the price, captured by β_{2ki}. Intuitively, if other investors' demands are very price-elastic, oligopolist j's price impact is smaller.

Finally, demand by the *competitive fringe investors* does not induce price movement and hence their optimization implies:

$$q_{0i} = \frac{\mu_{0i} - rp_i}{\rho\hat{\sigma}^2_{0i}}. \tag{5}$$

Private signals We assume that all active oligopolists ($j \in SA \cup LA$) observe their own private signals and then the price, which they use to update their beliefs without using additional information capacity. The choice of the vector of private signals $s_j = (s_{j1}, ..., s_{jn})$ about the vector of payoffs $z = (z_1, ..., z_n)$ is subject to a capacity constraint $I(z; s_j) \leq K_j$, where $I(z; s_j)$ quantifies the reduction in entropy of the payoffs conditional on the signals (defined below). For analytical tractability, we assume that the signals s_{ji} are independent across assets and investors. In this case, the total quantity of information obtained by an investor based on private signals is the sum of quantities of information obtained for each individual asset, $I(z_i, s_{ji})$. We can think of the information problem as a decomposition of each payoff into the signal component s_{ji} and a residual component δ_{ji} that represents the information loss because of the investor's capacity constraint. That is, $z_i = s_{ji} + \delta_{ji}$. If the signal and the residual are independent, then posterior beliefs are also normally distributed random variables. In particular, an active investor j's posterior beliefs about asset i's payoff, after observing the private signal, are distributed according to

$$z_i|s_{ji} \sim \mathcal{N}(\xi_{ji}, \eta^2_{ji}),$$

where the posterior mean and variance are given by Bayes' rule:

$$\xi_{ji} = \bar{z} + \frac{cov(z_i, s_{ji})}{var(s_{ji})}(s_{ji} - \bar{z}) = s_{ji},$$

$$\eta_{ji}^2 = \sigma_i^2 - \frac{cov^2(z_i, s_{ji})}{var(s_{ji})},$$

and $\bar{z}$ stands for the signal's unconditional mean. We define $\alpha_{ji} \equiv \frac{\sigma_i^2}{\eta_{ji}^2}$ to be an investor j's learning choice for each asset i. Given the private signal's structure, the information contained in a signal is given by

$$I(z_i, s_{ji}) = \frac{1}{2}\log\left(\prod_{i=1}^{n} \alpha_{ji}\right),$$

which gives rise to the capacity constraint:

$$\prod_{i=1}^{n} \alpha_{ji} \leq e^{2K_j},$$

Price signal All investors submit demand schedules that condition on the price p_i, which is equivalent to them observing the price explicitly. Therefore, as long as processing the information contained in the price does not take any information capacity, passive investors and the fringe will use the observation of the price, and update their beliefs according to Bayes' rule, which gives posterior beliefs with mean and variance given by:

$$\begin{aligned} \mu_{ji} &= s_{ji} + \frac{cov_j(z_i, p_i)}{var_j(p_i)}(p_i - E_j[p_i]), \\ \hat{\sigma}_{ji}^2 &= \frac{1}{\alpha_{ji}}\sigma_i^2 - \frac{cov_j^2(z_i, p_i)}{var_j(p_i)}. \end{aligned} \tag{6}$$

where the subscript j denotes the conditionality of each moment on the observed signal s_{ji}, which in the case of the competitive fringe is uninformative, and $\xi_{0i} = s_{0i} = \bar{z}$. Note that the update is investor specific, because after observing the private signals, the covariance, variance, and expectation of the price vary across investors.

Given (3) and (5), the demand schedule choices of investors, conditional on information choices $\{\alpha_{ji}\}_{i=1,\dots,n; j\in SA\cup LA}$, can be summarized by a fixed point $\{\beta_{0ji}, \beta_{1ji}, \beta_{2ji}\}_{i=1,\dots,n; j=0,\dots,l}$ of the system:[22]

$$\beta_{0ji} = \frac{-\frac{\gamma_{ji}}{\Delta_i}\left(-\bar{x}_i + \sum_{k=0}^{l}\lambda_k\beta_{0ki} + \sum_{k\neq j}\lambda_k\beta_{1ki}\frac{1}{\alpha_{ki}}\bar{z}\right)}{\rho\hat{\sigma}_{ji}^2 + r\frac{dp_i}{dq_{ji}}} \tag{7}$$

$$\beta_{1ji} = \frac{1-\frac{\gamma_{ji}}{\Delta_i}\left(\lambda_k\beta_{1ji} + \sum_{k\neq j}\lambda_k\beta_{1ki}(1-1/\alpha_{ki})\right)}{\rho\hat{\sigma}_{ji}^2 + r\frac{dp_i}{dq_{ji}}}, \tag{8}$$

$$\beta_{2ji} = \frac{1-\gamma_{ji}/r}{\rho\hat{\sigma}_{ji}^2 + r\frac{dp_i}{dq_{ji}}}, \tag{9}$$

where $\frac{dp_i}{dq_{ji}} = \frac{\lambda_j}{r\sum_{k\neq j}\lambda_k\beta_{2ki}}$ is the price impact of investor $j \geq 1$ on asset i, $\gamma_{ji} \equiv \frac{cov_j(z_i,p_i)}{var_j(p_i)}$ is used by agents to update their beliefs after observing the price using Bayes rule, and $\Delta_i \equiv r\sum_{j=0}^{l}\lambda_j\beta_{2ji}$ is the market sensitivity to the price weighted by size. Importantly, for the fringe, $\frac{dp_i}{dq_{0i}} = 0$, and for passive investors, the system simplifies to:

$$\beta_{0ji} = 0,$$
$$\beta_{1ji} = \beta_{2ji} = \frac{1}{\rho\hat{\sigma}_{ji}^2 + r\frac{dp_i}{dq_{ji}}},$$

since, without updating from prices, $\gamma_{ji} = 0$ for $j \in SP \cup LP$.

It is straightforward to see that all coefficients are decreasing in price impact (meaning that demand is lower as traders impact prices more). Further, as the overall market responds more to prices (higher levels of Δ_i), traders' demands increase as well. We present the details of the derivation of an oligopolist's utility and explicit maximization problem in Appendix A.5.[23]

3.1 Equilibrium

Denote $\bar{\alpha} = \{\alpha_{ji}\}_{i=1,...,n;j\in SA\cup LA}$ and $\bar{\alpha}_{-j} = \{\alpha_{ji}\}_{i=1,...,n;j\in SA\cup LA\setminus\{j\}}$. Let $\bar{\beta}(\bar{\alpha}) = \{\beta_{0ij},\beta_{1ij},\beta_{2ij}\}_{i=1,...,n;j=0,...,l}$ be a solution to (7)-(9) for a given information choice $\bar{\alpha}$.[24] For $i = 1,...,n$ and $j = 0,...,l$, an equilibrium consists of information and quantity choices of the fringe and oligopolists $\{\alpha_{ji}, q_{ji}\}$, the demand schedules choices $\bar{\beta}(\bar{\alpha})$, and price p_i, such that[25]

[22] We provide detailed derivations of the fixed point in Appendix A.4.

[23] We do not need to set up the ex-ante utility of the fringe investors since they make no information allocation choice. That is, $K_0 = 0$.

[24] In principle, there may be more than one $\bar{\beta}$ solution to the fixed point (7)-(9). However, in our numerical examples, we always find a unique positive solution.

[25] In the model, for any $\{\alpha_{ji}\}_{i=1,...,n;j=1,...,l}$, knowing $\{\beta_{0ij},\beta_{1ij},\beta_{2ij}\}_{i=1,...,n;j=0,...,l}$ is equivalent to knowing the price coefficients a_i, b_i, c_i, d_{ji}, $i = 1,...,n; j = 0,...,l$ such that $p_i = a_i + b_i\varepsilon_i - c_i\nu_i + \sum_j d_{ji}\zeta_{ji}$, where ζ_{ji} is the noise in the signal of oligopolist j about the payoff of asset i, so our solution is equivalent to the approach in Admati (1985).

1. $\alpha_{ji} = 1$ for $j \in SP \cup LP \cup \{0\}$, and for every $j \in SA \cup LA$, $\{\alpha_{ji}\}_{i=1,\ldots,n}$ maximizes ex-ante expectation of utility in (2), given $\bar{\beta}(\bar{\alpha})$ and $\bar{\alpha}_{-j}$. That is, each oligopolist's information choice is a best response to the other oligopolists' information choices (which are observable), while all the oligopolists internalize the effect of their information choices on the equilibrium behavior of everyone's quantities captured by β's .

2. $\bar{\beta}(\bar{\alpha})$ satisfies (7)-(9) for every feasible $\bar{\alpha}$ and $j \geq 0$. That is, given information choices and β of the other investors, each investor's quantity choices are optimal.

3. q_{ji} is given by (1), for all i and $j \geq 0$.

4. For all realizations of shocks z_i and private signals s_{ji}, the price p_i clears the market, for all i, that is,
$$\sum_{j=0}^{l} \lambda_j q_{ji} = x_i.$$

It is a known problem in the literature (e.g., Lambert, Ostrovsky, and Panov (2018)) that allowing for asymmetric strategies—in our case in learning and trading—introduces a significant level of complexity to the model, precluding analytical characterization of equilibrium existence. In our general setup, each oligopolist faces a different residual demand function that depends on other oligopolists' strategies, and chooses potentially different slopes of their demand schedules due to the fact that information is endogenously asymmetric as well. For that reason, we resort to characterizing the predictions of the general model numerically in Section 4, while at the same time verifying that the optimality conditions are satisfied with a very high numerical precision for all solutions.[26]

3.2 Price Informativeness

Following the work of Bai, Philippon, and Savov (2016), we define price informativeness as the covariance of the price with the fundamental shock, normalized by the variance of the price. Their analysis studies the empirical patters of Revelatory Price Efficiency (RPE), which is the exclusive

The one-to-one mapping between the price coefficients and the βs follows directly from equation (35).

[26] Our model has two layers of strategic interactions among oligopolists, in terms of their learning and trading strategies. Hence, uniqueness of equilibrium allocations is not guaranteed in general. In the parameterization we use in the numerical section, we find the learning strategy and best responses by always starting the solution algorithm from the largest oligopolist. We find that the algorithm finds the same solution independent of the initial guess. We also find that the allocation changes smoothly as we change parameters, suggesting that we are focusing on a single equilibrium outcome. More generally, we find a single equilibrium for a vast variety of empirically relevant parameter specifications.

outcome and focus of our theory. Theoretically, this measure maps well to our theory as the square root of the reduction in the variance of posterior beliefs of a Bayesian agent who learns from the price.[27] As Bai, Philippon, and Savov (2016) show, it can also be derived as a welfare measure using Q-theory. Given this definition, we can express price informativeness in our model as

$$PI_i = \frac{cov(p_i, z_i)}{\sigma_{pi}} = \frac{\sigma_i \sum_{j \in SA \cup LA} \omega_{ji} \frac{\alpha_{ji}-1}{\alpha_{ji}}}{\sqrt{\frac{\sigma_{xi}^2}{\sigma_i^2} + \left[\sum_{j \in SA \cup LA} \omega_{ji} \frac{\alpha_{ji}-1}{\alpha_{ji}}\right]^2 + \sum_{j \in SA \cup LA} \omega_{ij}^2 \frac{\alpha_{ji}-1}{\alpha_{ji}^2}}}, \tag{10}$$

where

$$\omega_{ji} \equiv \frac{\partial \lambda_j q_{ji}}{\partial s_{ji}} = \lambda_j \beta_{1ji}$$

is the responsiveness of an oligopolist's total demand for asset i to his private signal s_{ji}, which we term *the information pass-through*.

We note that price informativeness naturally only depends on the active investors' information and information pass-through choices. The endogenous terms ω_{ji} and α_{ji} enter the expression for price informativeness above intuitively. First, holding constant learning choices captured by α_{ji}, price informativeness is impacted by the degree to which demand choices of the oligopolists change in response to signals via the ω_{ji}s. If the oligopolists adjust their demand a lot in response to private signals (that is, they have high ω_{ji}s), price informativeness will increase due to higher covariance term $\omega_{ji} \frac{\alpha_{ji}-1}{\alpha_{ji}}$. At the same time, higher responsiveness of quantities to private signals means that any errors in the signals also show up in oligopolists' demand, which decreases price informativeness via the terms $\omega_{ij}^2 \frac{\alpha_{ji}-1}{\alpha_{ji}^2}$ in the denominator. These terms capture the noise in oligopolists' signals. In the numerical section, we isolate the effect of changing ω_{ji}s on price informativeness and refer to it as *the information pass-through channel*. Second, for a given information pass-through, price informativeness is affected by learning choices captured by α_{ji}s. They increase price informativeness by increasing the covariance of the price with the fundamental via the terms $\omega_{ji} \frac{\alpha_{ji}-1}{\alpha_{ji}}$, but also affect the noise in the price via the residual noise in private signals through the terms $\omega_{ij}^2 \frac{\alpha_{ji}-1}{\alpha_{ji}^2}$.[28] When we isolate the impact of learning choices on price informativeness, we refer to this effect as *the learning channel*.

[27] Bayes' rule implies, for prior variance σ_i, that the posterior variance is given by $\sigma_i^2 - \frac{cov^2(p_i, z_i)}{\sigma_{pi}^2} = \sigma_i^2 - PI_i^2$. For derivation of equation (10), see Appendix A.6.

[28] These terms are non-monotonic in α: increasing for $\alpha < 2$, then decreasing.

4 Quantitative Results

In this section, we provide a numerical characterization of the relationship between price informativeness and asset ownership structure. To isolate the pure effects of size, without the confounding effects of concentration, we first consider a monopolistic setup. Next, we show the impact of size in a more general setup with multiple oligopolists, some of which can be active and some passive. In this setting, we can also examine the role of ownership concentration.

The relative simplicity of our model prevents a fully fledged calibration exercise. Hence, for our simulations, we pick some parameter values with empirical targets in mind, while we normalize or pick others arbitrarily. For this reason, one should treat our results as a numerical illustration of our theoretical mechanism. To assess the robustness of such approach, we provide detailed and extensive sensitivity of our results to different specifications in Appendix B.4, which we also summarize in Section 4.8. The upshot of all these analyses is that the conclusions from our numerical exercises are robust to a variety of parameter and modeling specifications.

In our simulations, we choose parameters with two goals in mind: they have to be empirically relevant and the resulting solution needs to involve some degree of learning. We set some parameter values such that the benchmark model's median parameterization exhibits: aggregate oligopoly holdings of between 50% and 70% (the values corresponding to average stock-level institutional ownership in the last two decades), and market excess real return between 6.5 and 7% (the average over the 1980–2018 period). These targets depend on the size of the oligopolistic sector $\sum_{j=1}^{l} \lambda_j$,[29] the risk aversion coefficient ρ, and the informational capacity of large oligopolists, $K_j, j \in LA$, and small oligopolists, $K_j, j \in SA$. We assume that the small oligopolists' capacity is equal to 10% of that of the large oligopolists, that is, $K_k = 0.1K_j, j \in LA, k \in SA$. The risk-free rate is set to match 2.5% real return on 3-month T-bills. The rest of the parameters do not have empirical targets, so we set them arbitrarily and verify the robustness of our results to different choices. We set the payoff distribution to $\bar{z} = 10$ and $\sigma_i = 1$ for all i, the number of assets to $n = 5$, and the number of oligopolists to $l = 20$. Among oligopolists, half are assumed active and half passive. Specifically, we set the number of large active and passive oligopolists to 2, $LA = \{1, 2\}$ and $LP = \{3, 4\}$. We further assume 8 small active oligopolists, $SA = \{5, ..., 12\}$ and 8 small passive oligopolists $SP = \{13, ..., 20\}$. Risky assets are heterogeneous in their supply size, $\bar{x}_i$, which we interpolate

[29] We require the market return and ownership statistic to be satisfied for the midpoint of sector size in the size experiment.

linearly between 1 and 8, and in their volatility of supply, σ_{xi}^2, for which we target a coefficient of variation of 0.2, for all i.We allow λ_0 and $\{\lambda_j\}_{j=1}^{l}$ to vary across experiments and discuss these choices below. We summarize the parameter values in Table 1.

Table 1: Parameter values.

Parameter	Symbol	Value
Mean payoff	$\bar{z}_i$	10
Supply	$\bar{x}_i$	$\in [1, 8]$, linear distribution across i
Number of assets, oligopolists	n, l	$5, 20$
Risk-free rate	r	2.5%
Vol. of noise shocks	σ_{xi}	target coefficient of variation of 0.2 for all i
Vol. of asset payoffs	σ_i	1 for all i
Risk aversion	ρ	2.32
Information capacities	K_j	3.5 for $j \in BA$ and 0.35 for $j \in SA$
Investor masses	$\lambda_0, \{\lambda_j\}_{j=1}^{l}$	depending on experiment

The simulations generate equilibrium levels of price informativeness, and oligopolists' holdings for each asset. In our experiments, we use λs as proxies for stock ownership shares. We study the effects of different market structures on average price informativeness. We present the average and the cross-section of price informativeness, as well as a decomposition of the overall effect into the *learning channel*, implied by endogenously changing α_{ji}s in response to different market structures, and the *information pass-through channel*, implied by changing ω_{ji}s, consistent with equation (10). Notably, in our main experiments we do not change the aggregate amount of information in the economy (the maximum quality and number of signals that investors receive do not change), which means that all our effects are solely due to changing information choices arising from different market structures.[30]

4.1 The Effect of Size: Monopoly

We start by analyzing the effects of size in a monopolistic setup, with $l = 1$. For the experiment, we vary the size of the monopolist, λ_1, between 0.1 and 0.99. Figures 4-6 summarize the results. In Figure 4, we present the learning (panel a) and information pass-through (panel b) choices of the monopolist. As a benchmark, it is helpful to remember that atomistic agents would use their entire capacity to learn about a single asset, with preference towards the asset with the largest supply,

[30]This assumption can be justified with the intuition that increasing a manager's assets under management will not increase the manager's capacity to collect information. In Section 4.8, we summarize the results of an alternative setting in which managers can increase their capacity, subject to a cost. The qualitative aspects of our results do not change. We provide details of this experiment in Appendix B.3.

ceteris paribus.[31] For a monopolist, however, price impact implies that his demand function is concave in learning, which means that, due to decreasing marginal benefits, it is not optimal to focus his learning on only one asset. Instead, as the monopolist's size grows, it is optimal for him to learn about an increasing number of assets, reducing learning about high-supply assets (assets 4 and 5), and increasing learning about lower-supply assets (assets 2 and 3).[32] Thus, the larger the monopolist, and the higher the price impact, the more he spreads his learning across assets, increasing the informational content of prices of smaller assets and reducing the informational content of prices of larger assets.[33]

At the same time, increasing a monopolist's size has a non-monotonic impact on information pass-through. This effect is due to two opposing forces, reflected in ω_{ji} in equation (10). First, a larger monopolist has more assets under management, which increases the information pass-through for a given sensitivity of his demand to signals (captured by β_{1ji}). Second, a larger monopolist has larger price impact, which reduces the benefit of trading on information and implies that it is optimal to endogenously reduce the sensitivity of demand to signals, reducing information pass-through. When a monopolist is atomistic, his information pass-through is zero because of the negligible size; when a monopolist controls all assets in the market, pass-through is also zero because the monopolist's price impact is infinite. Anywhere in between, the pass-through is positive; hence, the tradeoff between the two forces leads to a hump shape in size, as can be seen in panel b.[34]

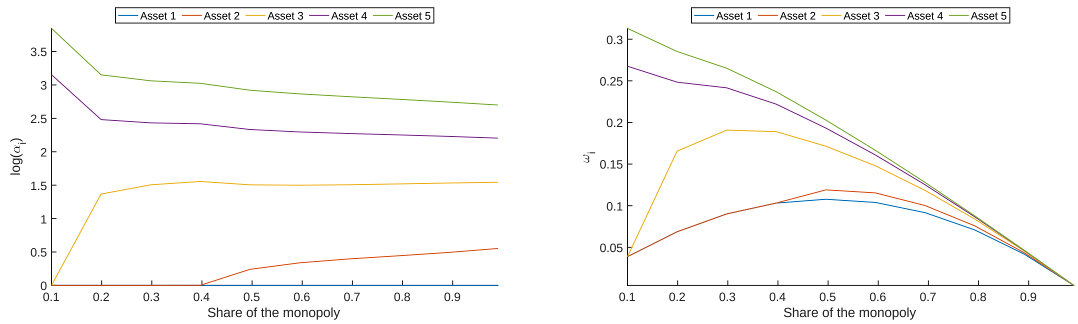
We present the net effect of the learning choices and information pass-through on price informativeness in Figure 5. Price informativeness of an asset is positive if and only if learning ($\log(\alpha_i)$) is positive, but the shape of the response to a growing size is strongly related to the hump shape of the information pass-through. In Figure 6, we decompose the impact of learning and information pass-through on average price informativeness. The results of the decomposition show that the information pass-through channel is quantitatively responsible for the hump shape of the average price informativeness. When we fix the information pass-through to the value corresponding to a large size of the monopoly, the values of price informativeness go down and the relationship with size loses its hump shape, due to the fact that a larger size leads to a smaller information

[31] In equilibrium, due to a standard water-filling argument, aggregate learning would be spread across multiple assets. For a detailed discussion of the competitive equilibrium with multiple assets, see Kacperczyk, Nosal, and Stevens (2019).

[32] In this experiment, the monopolist would still not learn about the smallest asset 1.

[33] We demonstrate these effects analytically in a simplified version of the monopoly setting in Appendix A.2, Proposition 2.

[34] We demonstrate the hump-shape of the price informativeness away from the $0, 1$ size limit in a simplified version of the monopoly setting in Appendix A.2, Proposition 3.



(a) Learning choices $\log(\alpha_i)$ by asset

(b) Information pass-through ω_i by asset

Figure 4: Decomposition of price informativeness and the monopoly size. The figure shows information choices α (panel a) and information pass-through ω (panel b), as a function of the monopoly size λ_1, for five assets that differ in their supply.

pass-through.

The effect due to the learning channel is smaller but quantitatively significant. Notably, pure reallocation of learning implies an increase in price informativeness as size of the monopolist increases. This is because larger size implies more diversification of learning (Figure 4) and an increase in average price informativeness, as information is now produced about a larger set of assets. Graphically, we observe that the fixed-learning line corresponding to a large monopolist's size lies above the line demonstrating the total effect of the two channels.

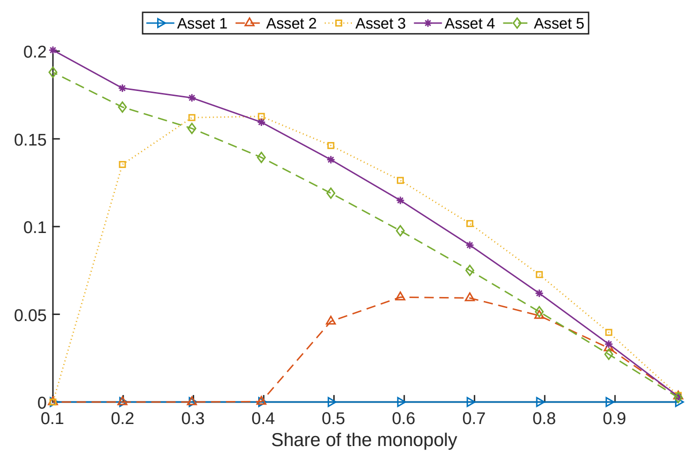


Figure 5: Price informativeness by asset and the monopoly size. The figure shows price informativeness for each asset in the model, ranked by supply $\bar{x}_i$ from lowest (asset 1) to highest (asset 5), as a function of the monopoly size λ_1.

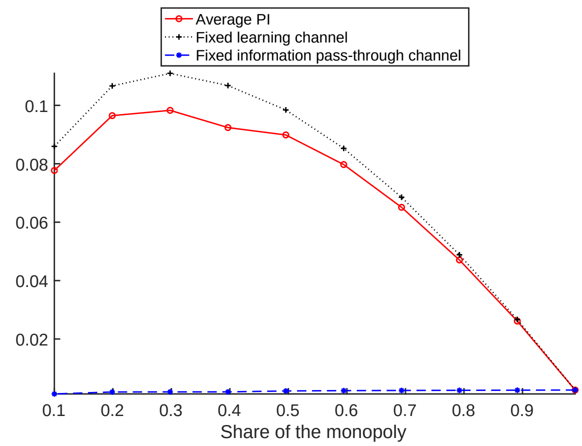


Figure 6: Decomposition of price informativeness and the monopoly size. The figure shows average price informativeness across five assets with different supply, as well as its decomposition. The 'Fixed learning channel' line is generated by holding α_{ji} fixed at values from the highest-share solution in equation (10). The 'Fixed information pass-through' line is generated by holding ω_{ji} fixed at values from the highest-share solution in equation (10). Share of a monopoly is given by λ_1.

The dominant role of information pass-through in driving the hump shape in price informativeness in the monopolistic case can also be demonstrated analytically. In this case, the price impact term simplifies to $\frac{dp_i}{dq_{ji}} = \frac{\lambda_1}{r(1-\lambda_1)\beta_{20i}}$, and the β terms of the monopolist's demand schedule are $\beta_{01i} = 0$, and $\beta_{11i} = \beta_{21i} \equiv \beta_i$, as there is no additional information in the price over and above that coming from the private signal of the monopolist (hence, the monopolist does not update from the price: $\gamma_{1i} = 0$). When we simplify the notation and denote the monopolist's learning choice as α_i, we can express price informativeness as:

$$PI_i = \frac{cov(p_i, z_i)}{\sigma_{pi}} = \frac{\sigma_i \lambda_1 \beta_i \frac{\alpha_i - 1}{\alpha_i}}{\sqrt{\frac{\sigma_{xi}^2}{\sigma_i^2} + \left[\lambda_1 \beta_i \frac{\alpha_i - 1}{\alpha_i}\right]^2 + \lambda_1^2 \beta_i^2 \frac{\alpha_i - 1}{\alpha_i^2}}},$$

where the information pass-through term, $\lambda_1\beta_i$, is given by

$$\lambda_1 \beta_i = \frac{\lambda_1(1-\lambda_1)\beta_{20i}\alpha_i}{\rho(1-\lambda_1)\sigma_i^2\beta_{20i} + \lambda_1\alpha_i}.$$

In equilibrium, the information pass-through is always non-negative, but non-monotonic, converging to zero as λ_1 tends to the boundary values of 0 or 1, that is,[35]

$$\lim_{\lambda_1 \to \{0,1\}} PI_i = 0. \tag{11}$$

This implies that the non-monotonicity of information pass-through is going to contribute to the potential non-monotonicity of price informativeness on an asset-by-asset basis, and quantitatively it can contribute to the non-monotonicity of the average price informativeness. Further, for very small or very large λ_1, information pass-through is the only determinant of the shape of price informativeness, as it converges to zero independent of the monopolist's learning choices. This result stands in contrast to the competitive case, discussed in Appendix A.1, in which price informativeness is strictly monotonic in size. We also demonstrate these forces away from the two limits in a simplified monopolistic setup in Appendix A.2 and summarize them in Section 4.8.

4.2 The Effect of Size: Oligopoly

In this section, we present the size experiment in the general, oligopolistic model. We examine the response of price informativeness to changes in the total size of the oligopoly sector, $\sum_{j=1}^{l} \lambda_j \equiv 1-\lambda_0$, while holding fixed the concentration of the sector. We consider both the aggregate and cross-sectional results. For our experiments, we require a baseline ownership structure of the oligopoly sector. For that, we proceed in two steps. First, we fix the *relative* distribution of sizes of the oligopolists. Within the small active and small passive group, we set relative sizes to be linearly distributed between 1 and 5. That is, the largest small active oligopolist is five times larger than the smallest one; the same is true for small passive oligopolists. Within large passive and active oligopolists, we set the largest active oligopolist to be three times larger than the second largest one; the same assumption we make for for the two large passive oligopolists. Having set the relative sizes, we then proceed to the second step of setting the *levels* of the size distribution. Specifically, we rescale the sizes so that the size of passive sector ($j \in SP \cup LP$) is 20% of total institutional size, and the size of small oligopolists ($j \in SA \cup SP$) is 10% of total institutional size. These properties result in a high concentration of institutional ownership and generate size of the passive sector consistent with the data we observe in late 2000s. Next, we solve the model with respect to different values of total size of oligopolistic sector relative to the market by varying $1 - \lambda_0$ between 20% (small oligopoly sector) and 99% (large oligopoly sector).

[35] For detailed derivation, see Appendix A.7.

In Figure 7, we present the relationship between the size of the oligopoly sector and average price informativeness. In the figure, each point corresponds to one solution of the model. The results bear a striking similarity to those in Figure 6: price informativeness exhibits a hump-shaped relationship with the sector size. We also note one exception. In the oligopolistic case, fixing the learning channel at the value corresponding to the large oligopolistic size actually lowers overall price informativeness, as opposed to increasing it, as it is the case in the monopolistic case. This difference arises from the presence of strategic interactions among oligopolists. Due to strategic substitutability in learning, each of the two large active oligopolists wants to learn about a different subset of assets. As the oligopolists grow in size, their ability to diversify their learning, as the monopolist did, comes at a sacrifice of the benefit from learning about mutually distinct subsets of assets. At their largest sizes, oligopolists prioritize learning about different subsets of assets over diversifying their learning, meaning that the learning channel actually lowers average PI relative to the case with smaller sectoral sizes.[36]

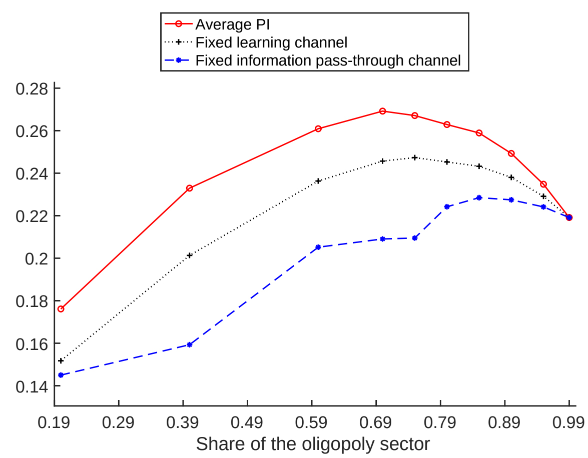


Figure 7: Decomposition of price informativeness and the oligopoly sector size. The figure plots average price informativeness across assets, as well as its decomposition. The 'Fixed learning channel' line is generated by holding α_{ji} fixed at values from the highest share solution in equation (10). The 'Fixed information pass-through' line is generated by holding ω_{ji} fixed at values from the highest share solution in equation (10). Share of oligopoly sector is given by $\sum_{j=1}^{l} \lambda_j$.

In Figure 8, we further show the results of our size experiment for individual assets that differ in terms of their supply. The driving force in the cross-section of assets is the interplay between the extensive and intensive margins of learning. On the extensive margin, since the marginal benefit

[36] This effect can be seen from the learning choices of the two largest oligopolists in the size experiment, presented in Figure 31 in Appendix B.7.

of learning is increasing in asset supply, there is a clear preference: high-supply assets are learned about first. On the intensive margin, all assets that are learned about are subject to similar effects we discussed above for the average size effects. The strength of each effect determines the ultimate relationship between size and price informativeness. For our simulation, high-supply assets (assets $4-5$) observe a humped, mostly downward-sloping shape between size and price informativeness, the result driven by information pass-through. In turn, low-supply assets (assets $2-3$) are affected more by the extensive margin of α_{ji}s. As the assets enter the pool of assets that are learned about, their price informativeness increases significantly. At the same time, as some oligopolists start learning about these assets, they commit less capacity to the high-supply assets, which exacerbates the drop in those assets' price informativeness.

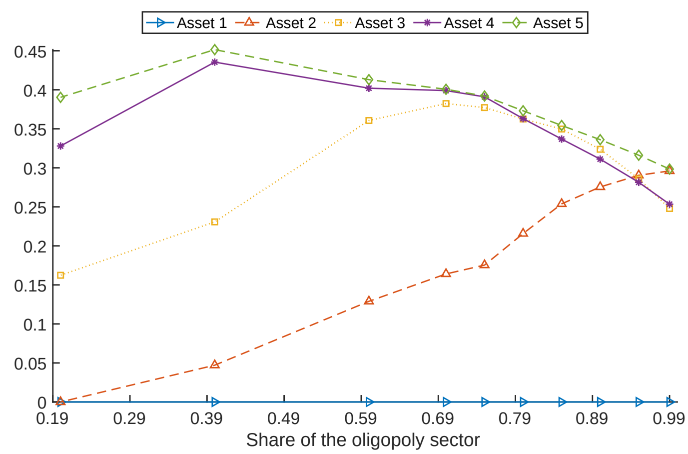


Figure 8: Asset-level price informativeness and the oligopoly sector size. The figure plots price informativeness for selected assets in the model, ranked by supply $\bar{x}_i$ from lowest (asset 1) to highest (asset 5), as a function of the share of oligopoly sector, given by $\sum_{j=1}^{l} \lambda_j$.

4.3 The Role of Passive Investors

The distinct importance of oligopolistic traders comes from two sources: their informational advantage and their large size. While all oligopolists exert price impact, not all of them use informationally intensive trading strategies. We model passive investors as being price-sensitive but not price-information-sensitive and as those who do not have any information capacity of their own, consistent with Grossman and Stiglitz (1980).[37] In particular, such passive investors can

[37]Allowing for passive investors to learn from prices (for free) does not qualitatively change the results of any of our experiments. We also note that there could be other ways to define passive investors, but we believe the two

hold benchmark portfolios (such as index funds or ETFs) but they can also be 'closet indexers', whose portfolios are well diversified (not market portfolios) but have little exposure to information (e.g., Cremers, Ferreira, Matos, and Starks (2016)). Our baseline model does not make a specific distinction of the two types of passive investors, but we provide the results using a more narrow definition of passive investors in Section 4.8.

To explore the consequences of this tradeoff for price informativeness, we conduct an experiment in which we vary the size of the passive sector (large and small investors combined) from 1% to 70% of the total oligopolistic size. The size distribution of $\{\lambda_j\}$s is set up the same way as that for the size experiment in Section 4.2, except that here we vary the passive size and set λ_0 equal to the midpoint value of that experiment of 0.4. We present the results for averages and the cross-section of assets in Figures 9 and 10, respectively.

We document several novel results. First, PI is concave and generally decreases with the size of the passive sector: As the active sector's size shrinks, the information pass-through of the large active oligopolists initially goes up and then decreases, while for the small active oligopolists, it decreases, which results in the shape of price informativeness, consistent with the intuition developed in previous sections.[38] This effect is supported by the counterfactual price informativeness in which learning channel is fixed. In Figure 9, we observe that the shape of price informativeness for that counterfactual preserves the slight hump shape. Second, as the active sector is getting smaller, active oligopolists become more specialized in learning, which reduces the number of assets they learn about.[39] This specialization process implies that price informativeness of the high-supply assets is relatively flat and hump-shaped and price informativeness of the low-supply assets is reduced, the results we show in Figure 10.

The cross-sectional patterns of price informativeness, illustrated in Figure 10, are particularly noteworthy. Specifically, the response of price informativeness to the growth of passive investors' share is heterogeneous across assets. Price informativeness of large-supply assets (e.g., assets 4 and 5) goes up or is flat as the active sector specializes in learning about those, while price informativeness of small-supply assets (e.g., assets 2 and 3) decreases. This result is reminiscent of a similar heterogeneity documented empirically in Bai, Philippon, and Savov (2016) and Farboodi

characteristics consistent across the definitions are that passive investors do not produce information, and that they care about price impact when trading. If these two characteristics are present, other formulations of passive investors (being able to learn from prices, buying market shares, etc.) should preserve our results.

[38] Information pass-through for the two largest oligopolists for this experiment is presented in Figure 32 in Appendix B.7.

[39] Learning choices for the two largest oligopolists for this experiment are presented in Figure 33 in Appendix B.7.

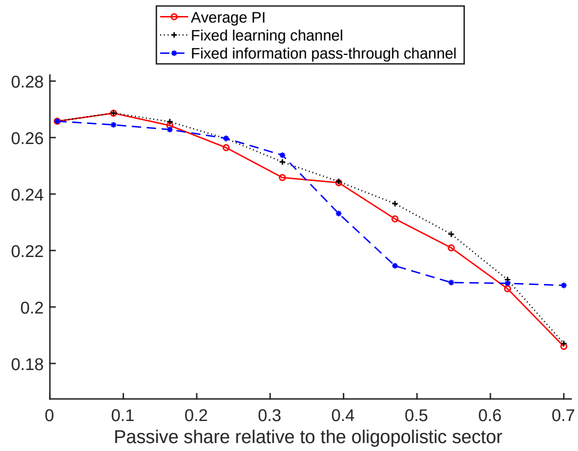


Figure 9: Price informativeness decomposition and the size of passive sector. The figure shows average price informativeness across assets, as well as its decomposition into the fixed learning and fixed pass-through channels. The 'Fixed learning channel' line is generated by holding α_{ji} fixed at values from the lowest size of the passive sector solution in equation (10). The 'Fixed information pass-through' line is generated by holding ω_{ji} fixed at values from the lowest size of the passive sector solution in equation (10).

et al. (2020). Through our framework, we are able to connect these phenomena to the growth of passive investing.

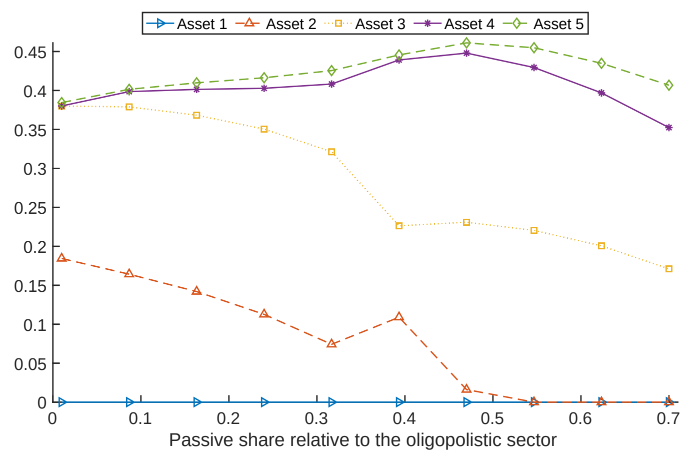


Figure 10: Asset-level price informativeness and the size of passive sector. The figure plots price informativeness for selected assets in the model, ranked by supply $\bar{x}_i$ from lowest (asset 1) to highest (asset 5), as a function of size of the passive sector relative to that of the overall oligopolistic sector.

4.4 Concentration of the Oligopoly Sector

The benefit of our oligopolistic setting is that one can study responses to changes in the concentration of the oligopoly sector, holding the sector size fixed. To study the effects of concentration in the model, we set the total oligopolistic sector size to the midpoint value from the size experiment ($\lambda_0 = 0.4$), as in the passive experiment. We then follow the parameterization strategy of the size experiment, generating different concentration levels by varying two values. First, we change the relative size of the two large active and two large passive oligopolists by varying λ_1/λ_2 and λ_3/λ_4 from 1.1 to 10. At the same time, we vary the relative size of the small sector, $\sum_{j\in SA\cup SP}\lambda_j/\sum_{j=1}^{l}\lambda_j$, from 10% to 3%. This experiment generates an increasing HHI index for oligopolistic ownership. Figure 11 presents average PI, as well as its decomposition in which we alternately fix the learning channel (α_{ji}) and the information pass-through (ω_{ji}) channel at the levels implied by the lowest-concentration scenario.

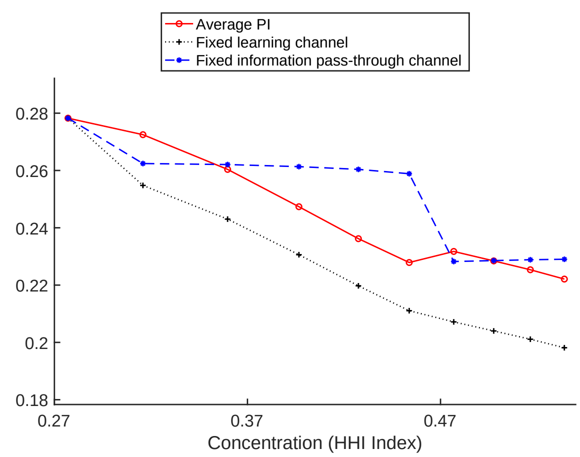


Figure 11: Average price informativeness decomposition and the size concentration. The figure plots average price informativeness across assets, as well as its decomposition. The 'Fixed learning channel' line is generated by holding α_{ji} fixed at values from the lowest concentration solution in equation (10). The 'Fixed information pass-through' line is generated by holding ω_{ji} fixed at values from the lowest concentration solution in equation (10). Size concentration is given by HHI index of oligopolistic shares, that is, $\sum_{j=1}^{l}(\frac{\lambda_j}{\sum_{k=1}^{l}\lambda_k})^2$.

The figure shows that the average price informativeness is decreasing in sector concentration. Even though the degree of the drop in price informativeness varies when holding either the learning or the information pass-through fixed, both effects work in the same direction (in both cases, price informativeness decreases with concentration). Like in the size experiment, the quantitative impact of the information pass-through channel is more significant. This logic is evident from the scenario in which we fix the degree of learning: changes in information pass-through alone imply an even larger drop in price informativeness as we increase concentration. For the scenario in which information pass-through is implied by the low concentration level, our analysis also indicates a decreasing price informativeness, but initially to a lesser degree. Intuitively, the underlying economic mechanism guiding the result is the reallocation of learning and information pass-through due to polarization of sizes. As the large oligopolists get larger, the active large oligopolists keep their learning diversified, which increases average PI. At the same time, lowering their information pass-through hurts average PI. As the smaller large active oligopolist gets smaller, he specializes his learning, and decreases information pass-through. Both effects decrease average PI. The difference between average PI and the PI implied by fixed information pass-through indicates that more diversified learning by the larger oligopolists initially dominates the more specialized learning by the smaller oligopolists, and that the dropping information pass-through of the large oligopolists is quantitatively important in driving the average price informativeness down.[40]

In Figure 12, we additionally present the overall impact of concentration on price informativeness on an asset-by-asset basis. We can see that, within our parameterization, the aggregate effect comes from individual price informativeness decreasing for most assets rather than from changes in the number of assets that are learned about. This is because the number of assets learned about largely depends on the total size of the oligopolistic market—a variable that we keep constant in this experiment. Note that, in this experiment, the extensive margin of learning is still operational at an individual oligopolist level. However, as Figure 12 demonstrates, across the different concentration levels, at least one oligopolist is learning about a constant set of assets.

4.5 The Combined Effect of Size and Passive Ownership

In previous sections, we presented the underlying mechanisms of the model in a framework in which one modification of the market structure was introduced at a time. However, as we observed in Section 2, the growth in size of the oligopoly sector and the growth in passive share within this sector are the two major changes that occurred concurrently over time. In this section, we present numerical results from the benchmark model in which we simultaneously introduce growth in size and in passive share. Specifically, as we increase size of the entire sector from 20% to 99%, we also increase the size of the passive sector relative to size of the entire oligopolistic sector, from 1% to 50%. We report the results in Figures 13 and 14. On average, price informativeness is hump

[40] These effects can be seen in Figure 34 in Appendix B.7.

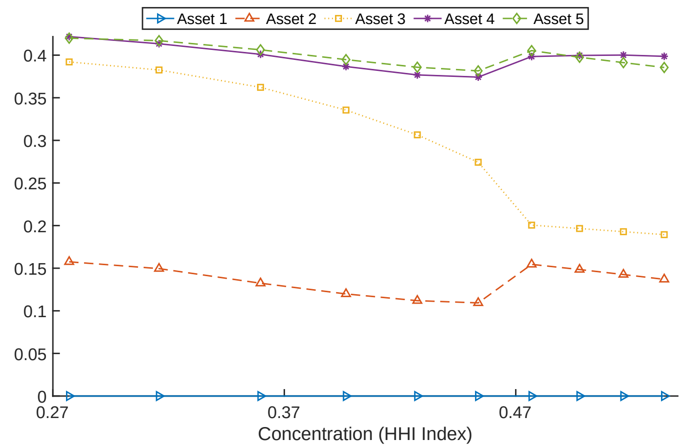


Figure 12: Asset-level price informativeness and the size concentration. The figure plots price informativeness for selected assets in the model, ranked by supply $\bar{x}_i$ from lowest (asset 1) to highest (asset 10), depending on size concentration, given by the HHI index of oligopolistic shares, i.e. $\sum_{j=1}^{l}(\frac{\lambda_j}{\sum_{k=1}^{l}\lambda_k})^2$.

shaped in size of the entire sector, with the peak appearing at lower values than what we observed for the *pure* size experiment, as the decline in price informativeness is further amplified by growth of investors with zero capacity. This can be clearly seen from the 'Fixed information pass-through' line, which still shows a slight hump shape driven by passive growth. Looking at the results on an asset-by-asset level in Figure 14, we can see a flat price informativeness curve for large assets (4 and 5) with large institutional ownership, and a hump shape for smaller assets. Overall, a pattern emerges of large cross-sectional differences in price informativeness across assets, but the pattern is much more muted from the time-series (x-axis) perspective. This result corresponds closely to the empirical findings in Bai et al. (2016) in which they look at PI of stocks sorted by institutional ownership.

4.6 The Role of Endogenous Learning

One of the novelties of our framework is that it features endogenous information choices together with quantity choices. The contrasting model with a fixed information structure is similar in spirit to Kyle (1989), in that the effect of market power on price informativeness depends entirely on the adjustment of quantities. To demonstrate the importance of modeling endogenous learning choices, we present the results from our three experiments—size, concentration, and the active/passive split—for the benchmark and exogenous information models. In the exogenous infor-

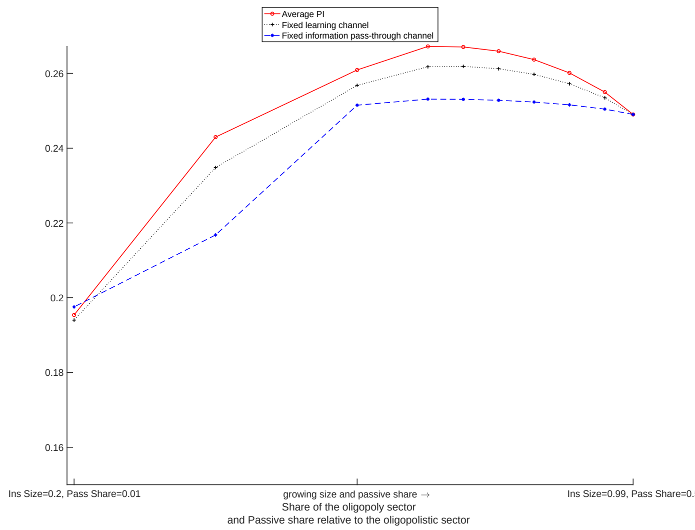


Figure 13: Average price informativeness decomposition and the size concentration. The figure plots average price informativeness across assets, as well as its decomposition. The 'Fixed learning channel' line is generated by holding α_{ji} fixed at values from the lowest concentration solution in equation (10). The 'Fixed information pass-through' line is generated by holding ω_{ji} fixed at values from the lowest concentration solution in equation (10). Size concentration is given by $\frac{\lambda_1}{\sum_{j=2}^{l} \lambda_j}$.

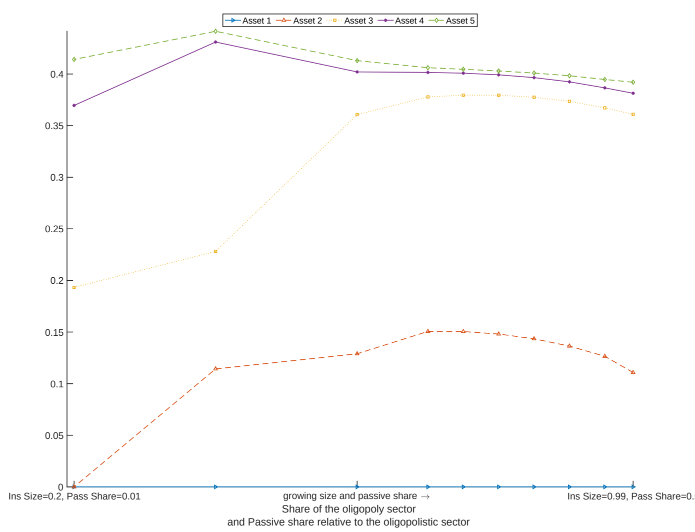


Figure 14: Asset-level price informativeness and the size concentration. The figure plots price informativeness for selected assets in the model, ranked by supply $\bar{x}_i$ from lowest (asset 1) to highest (asset 5), depending on size concentration, given by $\frac{\lambda_1}{\sum_{j=2}^{l} \lambda_j}$.

mation case, we endow oligopolists with the α_{ji} choices that are solutions to the benchmark model for one of the parameterizations in each experiment, and eliminate the possibility of re-optimization along the learning dimension. The response of the fixed-information model is driven entirely by information pass-through and hence the intuition is similar to our discussion of the fixed learning channel decomposition for the full model. Noteworthy, the information pass-through response is endogenously influenced by the information choices, and therefore the exogenous information results in this section and the fixed learning channel results in the previous sections are qualitatively and quantitatively different.

Figure 15 presents the results for the size experiment. Depending on the choice of α_{ji} (case 1 or 2), the maxima of the fixed-information model lie to the left or right of that of the benchmark model. Hence, the specific choice of the information structure significantly impacts the size of the oligopoly sector that maximizes price informativeness. In particular, for the exogenous information cases, the maxima are at 75% and 80%. Compared to the 70% maximum for the benchmark model, fixing the information structure can lead to an understatement or overstatement the optimal oligopoly share relative to the endogenous information model.

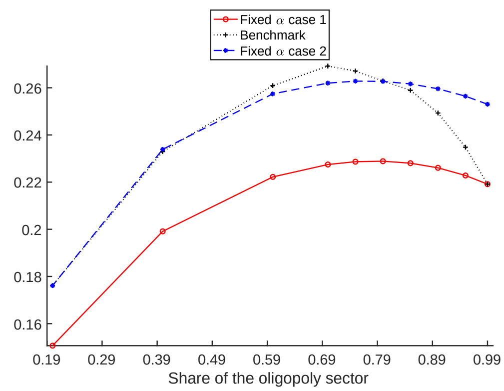


Figure 15: Price informativeness with endogenous (benchmark) and exogenous (fixed alpha) information choice as a function of the oligopoly sector share of the market. The figure plots average price informativeness across assets. The 'Fixed alpha case 1' line is generated by exogenously endowing investors with α_{ji} fixed at values from the lowest share solution of the benchmark model. The 'Fixed alpha case 2' line is generated by exogenously endowing investors with α_{ji} fixed at values from the highest share solution of the benchmark model. Share of oligopoly sector is given by $\sum_{j=1}^{l} \lambda_j$.

The experiment on the growth of the passive investor (Figure 16) provides another stark set of conclusions. For both choices of the exogenous information structure, the minimum passive share maximizes price informativeness, while in the benchmark model a 10% ownership is optimal from that perspective.

Finally, we present comparisons for the concentration experiment in Figure 17. In this case, the conclusions are as stark as for the other experiments. For one specification of exogenous learning in

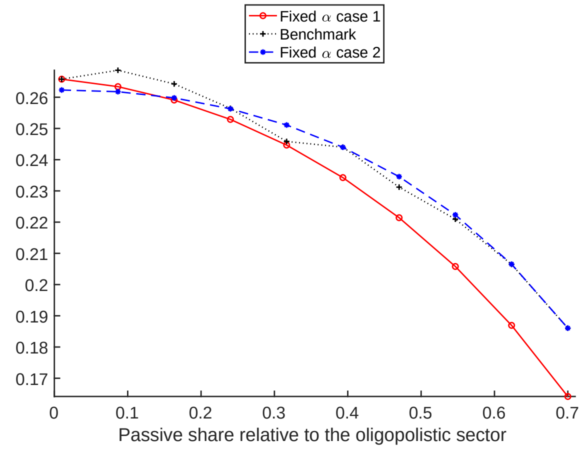


Figure 16: Price informativeness with endogenous (benchmark) and exogenous (fixed alpha) information choice as a function of passive sector's size relative to the entire oligopolistic sector. The 'Fixed alpha case 1' line is generated by exogenously endowing investors with α_{ji} fixed at values from the highest size of the passive sector solution of the benchmark model. The 'Fixed alpha case 2' line is generated by exogenously endowing investors with α_{ji} fixed at values from the lowest size of the passive sector solution of the benchmark model.

our counterfactual case 1, we have a maximum price informativeness at minimum concentration—just like the benchmark model. However, for a different exogenous learning choice in case 2, the concentration that maximizes price informativeness has a strictly higher, interior value.

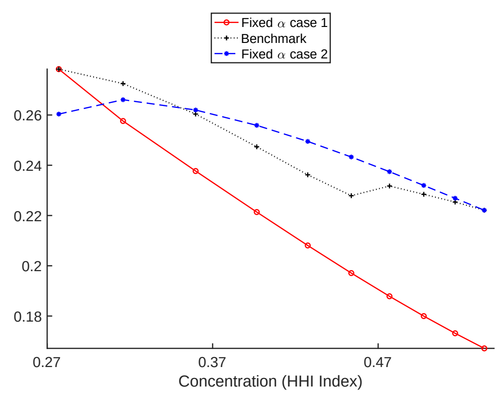


Figure 17: Price informativeness with endogenous (benchmark) and exogenous (fixed alpha) information choice as a function of size concentration. The figure plots average price informativeness across assets. The 'Fixed alpha case 1' line is generated by exogenously endowing investors with α_{ji} fixed at values from the lowest size concentration solution of the benchmark model. The 'Fixed alpha case 2' line is generated by exogenously endowing investors with α_{ji} fixed at values from the highest size concentration solution of the benchmark model. Size concentration is given by HHI index for institutional ownership.

In sum, depending on the specific choice of the exogenous information structure, the conclusions about the impact of size, passive ownership, or concentration on price informativeness can be dramatically different. Hence, considering a fully endogenous adjustment of quantities *and* information choices when studying price informativeness is crucial for drawing conclusions about the efficiency of different ownership structures.

4.7 Relation to Empirical Patterns

Our numerical results can be related to empirical patterns connecting asset ownership structure and price informativeness, as well as time-series properties of price informativeness, documented, for example, in Bai, Philippon, and Savov (2016). In our framework, price informativeness is due to investors' observing private signals, and thus can be interpreted as revelatory price efficiency (RPE). This notion of informativeness is also equivalent to forecasting price efficiency (FPE) since we consider no public signals.

Our first main result is that price informativeness, on average, is positively related to *total* institutional ownership for lower values of ownership and negatively related beyond a certain level of ownership. This is the outcome of the *size* experiment and of the *combined* experiment in this section. While we do not aim to pin down the specific point at which the slope of the relationship changes, *both* the increasing part and the decreasing part of of the slope is consistent with evidence in Bai et al. (2016) (their Table 2) who show that price informativeness has been going up over time since the 1960s, but has subsequently flattened or decreased post-2009, depending on the forecasting horizon. The results of our experiments, reported in Figures 7 and 13, are consistent with the data.

The second result of our model concerns the cross-section of assets. In our model, assets with large supply exhibit greater price informativeness, on average (cross-sectional Figures 8, 10, 12, and 14), with large assets also having higher institutional ownership in equilibrium. This result corresponds well to evidence in Figure 8 of Bai et al. (2016), which shows that assets with higher ownership have higher price informativeness. What is interesting to note in that figure is that the trend of price informativeness over time is actually flat, that is, the difference between assets in terms of their PI works more like a fixed effect. This result is surprising at first given our earlier discussion but can actually be rationalized if we factor in another force in both the model and the data, which is the increasing share of passive investors over time. If we take into account both trends in the data, as presented in Figure 14, we can see that the growing total institutional size is pushing price informativeness higher while the increase in passive sector's size is pushing it lower.

Consequently, our model can produce a stationary price informativeness across assets resulting from the net effect of the two trends we observe in the data.

Finally, our cross-sectional prediction on the decreasing relationship between asset ownership concentration and price informativeness is broadly consistent with empirical evidence in Massa et al. (2020) who show that as a result of the merger between two large institutional investors, Blackrock and BGI, liquidity and trading intensity go down, which likely implies lower price informativeness.

4.8 Sensitivity and Robustness

Below, we briefly discuss additional analytical and numerical results derived in the Appendix that provide more intuition behind our results and show the sensitivity of our findings to alternative model specifications.

Analytical Results from Simplified Limit Cases In order to provide additional intuition and show the robustness of our findings, we provide an analytical characterization of the results from our model for simplified limit cases. Specifically, we first study the relation between size and price informativeness for two polar opposites of our framework: competitive model and monopolistic model. Then, in a simplified duopolistic setup, we study the role of relative sizes for information and price informativeness, and investigate the interaction between passive and active oligopolists.

First, in the fully competitive setup in Appendix A.1, we show that price informativeness, asset-by-asset and on average, is strictly monotonic in the size of active investor sector. The intuition for this result is that, absent price impact considerations, the information pass-through of competitive investors is always the same (and normalizes to 1). As a result, price informativeness is solely driven by learning choices, which in the competitive setup are given by each investor spending all of their information capacity on a single asset. The larger is the mass of active investors, the more learning there is for each asset that is learned about. This result is summarized in Corollary 1.

Second, in Appendix A.2, we provide further analytical characterization of the monopolistic setup. Relative to the limit results in Section 4.1, we are able to characterize behavior away from the limits by using a simplified version of the monopolistic setup in which the fringe does not learn from prices. Thanks to that simplification, we can show that as the size of the monopolist increases, he chooses to learn about an increasing number of assets, ranked by the gain from learning, the quantity which is monotonically related to size (Proposition 2). Additionally, we show that price informativeness is strictly increasing in size when the monopoly size is sufficiently small and strictly decreasing if the size is sufficiently large. We additionally show that this monotonic behavior of price informativeness is driven by information pass-through of the monopolist (Proposition 3 and Corollary 2). These analytic results point to the robustness of our numerical results, as they hold independently of parameter choices.

Finally, in a duopolistic setup with exogenous quality of the signals in Appendix A.3, we analyze how relative sizes of the two duopolists affect price informativeness, depending on the relative quality of their signals. First, we find that symmetric sizes of the duopolists are only optimal if their information is symmetric. This confirms the strong relation between the ownership structure, information, and price informativeness shown in Section 4.6. Second, we show that even if one duopolist has no information (is passive), then the size of that duopolist that maximizes price informativeness is strictly larger than zero. This is due to the fact that the active duopolist may be 'too large' and lowering his size may increase information pass-through and hence increase price informativeness. This result provides more intuition for the slight hump shape of the average price informativeness for the passive experiment. Finally, we show in this case that, everything else equal, the active duopolist's information pass-through is always *smaller* if he faces a passive, rather than active, duopolist as a competitor. The intuition for that is that lower information quantity of the large passive competitor increases the price impact of the active competitor, reducing the benefit of acting on information. This illustrates an additional general equilibrium effect that acts in response to a rise of passive sector.

Sensitivity to Parameter Choices In order to check the sensitivity of our results to parameter choices, we conduct a variety of sensitivity tests to alternative parameter settings, which we report in Section B.4. Specifically, we vary the values of mean payoff $\bar{z}_i$, volatility of payoffs σ_i, the supply distribution $\{x_i\}_{i=1}^n$, and the capacity choices K_j. In each instance, we recalibrate the risk aversion parameter ρ in order to match the mean payoff for the mid-point of size, $\lambda_0 = 0.4$, just like in the benchmark calibration. The results are reported in Figures 24-27. The conclusions from our three experiment hold for each alternative parameter specification. The intuition for the robustness of our results derives from the analytical section A.2, where we show that the diversification in learning due to increased size and the non-monotonicity and importance of information pass-through holds in general, independently of the specific parameter choices.

DARA Specification In Appendix B.1, we present the results of our three experiments in a setup in which the coefficient of absolute risk aversion is investor-specific, and is negatively related to investor size.[41] We find that for a range slopes of this negative relationship, the main conclusions from our three experiments stay qualitatively the same (see Figures 18 and 19). Intuitively, price informativeness is determined by learning choices across assets and information pass-through, but decreasing absolute risk aversion affects demand for all assets and does not alter the relative benefit of learning about one asset versus another. As a result, introducing DARA does not appreciably alter our results.

Linear Entropy Constraint We treat our choice of information constraints as a natural one, as it is widely used in economics and appeals to the literature that axiomatically derives cost of information functions (see Cover and Thomas (2006), Sims (2003), and discussion in Van Nieuwerburgh and Veldkamp (2010)). However, in economic settings, one could definitely rationalize other cost structures, raising valid questions about how our results are affected by these choices. That is why, in Appendix B.2, we test the sensitivity of our results to an alternative specification of the entropy constraint. Specifically, we switch to a linear entropy constraint. In terms of the response of average price informativeness in our three main experiments, the linear entropy constraint model gives qualitatively similar results to the benchmark model (Figure 20). Quantitatively, the peak of the price informativeness curve for the size experiment occurs at higher values of the entire oligopoly sector size, and the passive and size experiments are very close to the benchmark predictions. Overall, the conclusions from the experiments carry the same message as the benchmark model.

Endogenous Capacity Choice As we show above, the incentives to learn about particular asset payoffs vary with the size distribution. Intuitively, the distribution could also affect incentives to invest in information-processing capacity K_j, given that price impact diminishes the rents from informed trading. In Appendix B.3, we consider an extension of the baseline model that allows for endogenous capacity choice subject to a convex cost. We show that for a variety of parameterizations of the cost curvature and its level, the variation in the optimal capacity choice by large traders is consistent with this intuition. However, quantitatively, optimal capacity choice is still relatively stable across market structures, and hence our conclusions derived from the baseline model with fixed capacity are not altered in a significant way. Intuitively, as a large traders' size varies, two

[41] Specifically, we assume that $\rho_j = \bar{\rho} - s\lambda_j$, where s is a slope parameter.

opposing forces shape optimal information capacity choice. On the one hand, larger size means that the information capacity will be applied to a larger size of the portfolio, implying economies of scale and an increase in optimal capacity. On the other hand, larger size means larger price impact and hence the rents from better information cannot be fully captured. The interaction of these opposing incentives implies that the variation in optimal capacity levels is small as the size distribution changes.

Model with Passive Indexers As we point out in Section 4.3, our baseline model does not make a specific distinction between passive investors that are pure indexers and hold the market portfolio, and 'closet indexers' whose portfolios are well diversified (not market portfolios) but have little exposure to information. In Appendix B.5, we present the results from the model in which we expand the type of passive investor to include both types. Specifically, for our three experiments, we parameterize the model such that half of the passive investors are passive indexers, which means that they hold the index, defined as an average supply-weighted portfolio, and are not price sensitive. That assumption essentially allocates a fraction of the overall supply of the asset to the passive indexers, effectively changing the average supply of the assets faced by the remaining investors. Figure 28 shows the results, which are qualitatively consistent with the benchmark specification. Intuitively, the main results of our model come from the effect of size of active investors relative to market and its impact on information allocation and pass-through; the feedback from passive investors to active investors is relatively modest.

Model with Informed Retail Investors Our benchmark model assumes that retail investors have no information capacity. However, evidence suggests that at least some of the investors may be informed. In Appendix B.6, we discuss a version of the model, in which we introduce small retail investors with positive capacity. Specifically, we parameterize the model to include 50 retail investors (on top of 20 oligopolists) whose capacity equals the capacity of the small active oligopolists. In the size experiment, the retail investors are accounted for as part of the non-institutional sector in the model. Hence, they introduce an additional effect of changing institutional ownership size, namely that increased institutional ownership reduces ownership of informed retail investors. We present results from two parameterizations, in which retail investors constitute 50% or 98% of the non-institutional sector (Figures 29 and 30).

The predictions of the model with retail informed investors differ slightly from the benchmark model but are close. This is due to several factors. First, the size of each retail investor is small, which means that they will not have high information pass-through due to their small economic significance. Also, their small size means they will endogenously specialize in learning about only one asset, which further diminishes their impact on price informativeness. Finally, shifting ownership from small oligopolists to retail investors should, in principle, have a very small effect on price informativeness, and so the primary difference for our results comes from shrinking large active oligopolists.

5 Concluding Remarks

Equities are overwhelmingly held by more sophisticated investors, and ownership is especially concentrated among the largest investors. This skewed ownership structure has triggered an active discussion among financial regulators and industry participants over its implications for welfare and financial stability. Proponents of regulation have argued in favor of reduced power for large institutions, while critics of such reforms argue that the information such institutions imbue into prices makes market concentration worthwhile. But most of the arguments are based on anecdotal evidence or are based on empirical data that could be subject to numerous explanations and do not necessarily provide clear out-of-sample insights. In the absence of a well-specified economic model, it is difficult to shed light on the argument and understand how to quantify the tradeoffs of a more concentrated marketplace.

This paper takes a step towards addressing this issue by developing a general equilibrium model in which asymmetric information, asymmetric market power, and asset heterogeneity are important determinants of the informational efficiency that regulators might want to maximize. While regulators' objective functions can take different forms, we believe that a setting in which the informational content of prices is of a planner's interest is appropriate to characterize the world of equity ownership.[42] Our theory makes a methodological contribution in generalizing models of asymmetric information (building on, for example, Kyle (1989)), by explicitly modeling information allocation in the presence of asymmetric market power and nontrivial heterogeneities across investors and assets.

Contrary to common wisdom, our results suggest that an intermediate size of the institutional

[42] Most analyses of price informativeness in the literature show that price informativeness enhances efficiency of markets (Vives (2011), Vives (2014), Rostek and Weretka (2012), and Lambert et al. (2018)). A notable exception to this result is Vives (2017).

sector maximizes the information contained in prices, even if large investors have superior information capacity. Our results confirm that for ownership levels equal to those currently found in the U.S., average price efficiency is positively related to the levels of large ownership but negatively related to its concentration. Further, we show that average price informativeness across assets can be maximized for admissible values of ownership and concentration. This result suggests that policy makers should consider concentration in addition to size when constructing policies to maximize price efficiency.

Our model applies to settings that involve a rich cross-section of assets, informational asymmetries across oligopolistic agents, and differences in market power. At a broad, policy level, the model can be also fruitfully used in discussions of market transparency and access to information. At the same time, the model naturally abstracts from other dimensions relevant for policy makers, such as investment costs or sectoral fund flows given that the size distribution is an input in our analysis. The model we propose is also static in nature and thus may not fully capture the dynamic aspects of trading, discussed in other research (e.g., Wang (1993)). To build a fully dynamic model of trading could require introduction of stochastic illiquidity costs. We think that incorporating such dynamics would amplify the trading patterns we document here; that is, investors may internalize even more the dynamic aspects of price impact, which could further depend on investor and asset-specific costs of trading. While these issues are clearly practically relevant, they require a more extended modeling framework which is beyond the scope of this paper.[43] We also abstract from endogenous changes in market structure due to entry and exit, which could change the aggregate amount of information in the economy. Another important aspect is the rise of 'big data' and the extent to which this has been harnessed by larger players. Finally, we omit any issues related to optimal asset management contracts. While each of these angles will provide alternate channels by which institutional investors could impact price efficiency, we feel that our model provides analysis of two of the most pressing: market power in trades, and the impact on learning decisions. We leave any other issues for future research.

[43] As a simplification, one could also introduce some basic intuition by comparing the results conditional on different parameter choices. These could be motivated by natural dynamics in the data, such as time-varying risk aversion or time-varying level/volatility of market returns. Such exercise would essentially build on our Section 4.8.

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A Appendix

A.1 Analytical characterization of price informativeness under perfect competition

Below, we present an analytical characterization of the impact of size on price informativeness in the perfectly competitive case. Specifically, we show that price informativeness grows monotonically with the size of informed investor sector.

Let all investors be perfectly competitive price takers, with fraction λ_1 having positive capacity $K > 0$, and fraction $\lambda_0 = 1 - \lambda_1$ having zero capacity. Both types of investors learn from prices without using informational capacity. We guess and later verify that agents choose one asset to spend all of their informational capacity on–meaning that each asset has two types of investors: informed investors who have spent all their capacity on that asset, and uninformed investors who have not. Investors solve the standard portfolio allocation problem, given their posterior beliefs, which results in optimal portfolio holdings given by:

$$q_{ji} = \frac{\mu_{ji} - rp_i}{\rho\hat{\sigma}_{ji}^2},\ j = 0, 1, \tag{12}$$

where μ_{ji} and $\hat{\sigma}_{ji}^2$ are the mean and variance of investors' posterior beliefs after observing their private signals (in case of informed investors) and the price, given by (6).
Given the optimal portfolio holdings as a function of posterior beliefs, the ex-ante optimal distribution of signals maximizes the ex-ante expected utility:

$$E_0[U_j] = \frac{1}{2\rho}\sum_{i=1}^{n}\frac{E_0\left(\mu_{ji} - rp_i\right)^2}{\hat{\sigma}_{ji}^2}, \tag{13}$$

where the choice of the vector of signals $s_j = (s_{j1}, ..., s_{jn})$ about the vector of payoffs $z = (z_1, ..., z_n)$ is subject to a capacity constraint $I(z; s_j) \leq K_j$. Following Admati (1985), we conjecture and later verify that prices are

$$p_i = a_i + b_i\varepsilon_i - c_i\nu_i, \tag{14}$$

where coefficients a_i, b_i, c_i are determined in equilibrium. Summarizing learning choices by $\alpha_{1i} \equiv \frac{\sigma_i^2}{\hat{\sigma}_{1i}^2}$, we can express the maximization problem of an informed investor as[44]

$$\max\sum_{i=1}^{n} G_i\alpha_{1i} \quad \text{subject to} \quad \frac{1}{2}\sum_{i=1}^{n}\log(\alpha_{1i}) \leq K_1,$$

where

$$G_i \equiv \frac{\left(\overline{z} - ra_i\right)^2}{\sigma_i^2} + \left(1 - rb_i\right)^2 + r^2c_i^2\frac{\sigma_{xi}^2}{\sigma_i^2}. \tag{15}$$

The linear objective function and concave functional form of the constraint implies that each competitive investor $j = 1$ specializes in learning only about one asset. For the remaining assets, that investor's holdings are determined by prior beliefs. In equilibrium, all assets that are learned about provide the same gain G_i, and all other assets offer strictly lower gains. The equilibrium of the competitive economy can be summarized by the mass of informed agents that learn about asset i, $\hat{\lambda}_i \geq 0, \forall i$, with $\sum_{i=1}^{n}\hat{\lambda}_i = \lambda_1$.

We can use the market clearing condition to derive the price coefficients and express G_i as a function of fundamentals and learning choices only, leading to Proposition 1, which states that investors have preference to learn about assets that are in large supply ($\bar{x}_i$) or are more volatile (σ_{xi}^2 or σ_i^2). Additionally, G_is depend on λ_1 only through $\hat{\lambda}_i$, and $dG_i/d\hat{\lambda}_i < 0$. As a consequence, and given $\sum_i\hat{\lambda}_i = \lambda_1$, we have (for proof, see Section A.1.2 below):

[44] For detailed derivation of (15), see Section A.1.1 below.

Proposition 1. *The following statements hold in equilibrium:*

1. *The shadow value of information, G_i, is increasing in $\bar{x}_i$, σ_{xi}^2, and σ_i^2.*
2. *For all assets $i = 1, ..., n$, $\frac{d\hat{\lambda}_i}{d\lambda_1} \geq 0$, with strict inequality for assets that are learned about.*

Price informativeness in the competitive model is given by:

$$PI_i = \frac{b_i \sigma_i^2}{\sqrt{b_i^2 \sigma_i^2 + c_i^2 \sigma_{xi}^2}} = \frac{\sigma_i^2}{\sqrt{\sigma_i^2 + (\frac{c_i}{b_i})^2 \sigma_{xi}^2}}.$$

Using the price coefficients derived in Appendix A.1.2, we have

$$\frac{c_i}{b_i} = \frac{\rho \sigma_i^2}{\hat{\lambda}_i (\alpha_i - 1)}.$$

Since all investors allocate their full capacity to a single asset, the average learning choice of investors learning about asset i, denoted α_i above, is equal to e^{2K_1}, and price informativeness is strictly monotonic in $\hat{\lambda}_i$. Hence, Proposition 1 implies:

Corollary 1. $\frac{dPI_i}{d\lambda_1} \geq 0 \, \forall i$, *with strict inequality if the asset is learned about ($\hat{\lambda}_i > 0$).*

Summarizing, in the competitive model, each individual asset's price informativeness is a strictly monotonic function of the size of the informed competitive investor sector, λ_1, and thus so is the average price informativeness.

A.1.1 Derivation of Equation (15)

In this section, we focus solely on the informed investors ($j = 1$). Their information choice solves

$$\max_{\{\hat{\sigma}_{ji}^2\}_{i=1}^n} U_0 \equiv \frac{1}{2\rho} \sum_{i=1}^n \frac{E_0 \left(\mu_{ji} - r p_i\right)^2}{\hat{\sigma}_{ji}^2} \tag{16}$$

subject to the relative entropy constraint

$$\prod_{i=1}^n \frac{\sigma_i^2}{\hat{\sigma}_{ji}^2} \leq e^{2K_1}. \tag{17}$$

Hence, the gain from learning about a particular asset is the same across all competitive investors. To derive the above, note that the objective is:

$$U_0 = \frac{1}{2\rho} \sum_{i=1}^n \frac{\widehat{R}_i^2 + \widehat{V}_i}{\hat{\sigma}_{ji}^2},$$

where

$$\widehat{R}_i \equiv E_0 \left(\mu_{ji} - r p_i\right) = \bar{z} - r E_0(p_i) = \bar{z} - r a_i,$$

and

$$\widehat{V}_i \equiv V_0 \left(\mu_{ji} - r p_i\right) = var\left(\mu_{ji}\right) + r^2 \sigma_{pi}^2 - 2r cov\left(\mu_{ji}, p_i\right).$$

Let α_{ji} denote the learning choice of the particular maximizing investor, while α_i the learning choice of all investors learning about asset i (which are of mass $\hat{\lambda}_i$). Given that each investor's solution is going to be a corner, in equilibrium $\alpha_{ji} = \alpha_i = e^{2K_1}$. With this notation in hand, we have:

$$var\left(\mu_{ji}\right) = var(s_{ji} + \frac{cov(z_i, p_i)}{\sigma_{pi}^2}(p_i - E_j[p_i])) = (1 - \frac{1}{\alpha_{ji}})\sigma_i^2 + \frac{cov^2(z_i, p_i)}{\sigma_{pi}^4}[var(p_i) + b_i^2(1 - \frac{1}{\alpha_i})\sigma_i^2]$$

$$+2\frac{cov(z_i,p_i)}{\sigma_{pi}^2}[cov(s_{ji},p_i)-cov(s_{ji},E(p_i))-\frac{cov(z_i,p_i)}{\sigma_{pi}^2}cov(p_i,E(p_i)]$$

$$=(1-\frac{1}{\alpha_i})\sigma_i^2+\frac{cov^2(z_i,p_i)}{\sigma_{pi}^4}[c^2\sigma_{xi}^2+b_i^2\frac{1}{\alpha_i}\sigma_i^2]$$

Given that the conditional variance of the price is: $c^2\sigma_{xi}^2+b_i^2\frac{1}{\alpha_i}\sigma_i^2$, we have

$$var(\mu_{ji})=(1-\frac{1}{\alpha_i})\sigma_i^2+\frac{cov^2(z_i,p_i)}{\sigma_{pi}^2}$$

Now for the covariance:

$$cov(\mu_{ji},p_i)=cov(s_{ji}+\frac{cov(z_i,p_i)}{\sigma_{pi}^2}(p_i-E_j[p_i]),p_i)=cov(s_{ji},p_i)+\frac{cov(z_i,p_i)}{\sigma_{pi}^2}var(p_i)-\frac{cov(z_i,p_i)}{\sigma_{pi}^2}cov(E(p_i),p_i)$$

$$=cov(s_{ji},p_i)+\frac{cov(z_i,p_i)}{\sigma_{pi}^2}[var(p_i)-b_i^2(1-\frac{1}{\alpha_i})\sigma_i^2]$$

$$=b_i(1-\frac{1}{\alpha_i})\sigma_i^2+cov(z_i,p_i)=b_i(1-\frac{1}{\alpha_i})\sigma_i^2+b\frac{1}{\alpha}\sigma_i^2=b\sigma_i^2$$

$$\hat{V} \quad = \quad (1-\frac{1}{\alpha_i})\sigma_i^2+\frac{cov^2(z_i,p_i)}{\sigma_{pi}^2}+r^2b^2\sigma_i^2+r^2c^2\sigma_{xi}^2-2rb\sigma^2$$

And the ex-ante expected utility is:

$$U_0 \quad = \quad \frac{1}{2\rho}\sum_i\left[(\bar{z}-ra_i)^2+\sigma_i^2+r^2b^2\sigma_i^2+r^2c^2\sigma_{xi}^2-2rb\sigma^2-\left[\frac{1}{\alpha_i}\sigma_i^2-\frac{cov^2(z_i,p_i)}{\sigma_{pi}^2}\right]\right]\frac{1}{\frac{1}{\alpha_{ji}}\sigma_i^2-\frac{cov^2(p_i,z_i)}{\sigma_{pi}^2}}$$

Which becomes

$$U_0 \quad = \quad \frac{1}{2\rho}\sum_i[(\bar{z}-ra_i)^2+\sigma_i^2+r^2b^2\sigma_i^2+r^2c^2\sigma_{xi}^2-2rb\sigma^2]\frac{1}{\frac{1}{\alpha_{ji}}\sigma_i^2-\frac{cov^2(p_i,z_i)}{\sigma_{pi}^2}}+const.$$

Consider now $\frac{1}{\alpha_{ji}}\sigma_i^2-\frac{cov^2(p_i,z_i)}{\sigma_{pi}^2}$. Given that $p_i=a_i+b_i\varepsilon_i-c\nu_i$, we have

$$\frac{1}{\alpha_{ji}}\sigma_i^2-\frac{cov^2(p_i,z_i)}{\sigma_{pi}^2}=\frac{1}{\alpha_{ji}}\sigma_i^2-\frac{b^2\frac{1}{\alpha_{ji}^2}\sigma_i^4}{b^2\frac{1}{\alpha_{ji}}\sigma_i^2+c^2\sigma_{xi}^2}$$

$$=\frac{\frac{1}{\alpha_{ji}}\sigma_i^2(b^2\frac{1}{\alpha_{ji}}\sigma_i^2+c^2\sigma_{xi}^2)-b^2\frac{1}{\alpha_{ji}^2}\sigma_i^4}{b^2\frac{1}{\alpha_{ji}}\sigma_i^2+c^2\sigma_{xi}^2}=\frac{\frac{1}{\alpha_{ji}}\sigma_i^2c^2\sigma_{xi}^2}{b^2\frac{1}{\alpha_{ji}}\sigma_i^2+c^2\sigma_{xi}^2}$$

Given that, the maximization problem becomes

$$
\begin{aligned}
U_0 &= \frac{1}{2\rho}\sum_i [(\bar{z}-ra_i)^2+\sigma_i^2+r^2b^2\sigma_i^2+r^2c^2\sigma_{xi}^2-2rb\sigma^2]\frac{b^2\frac{1}{\alpha_{ji}}\sigma_i^2+c^2\sigma_{xi}^2}{\frac{1}{\alpha_{ji}}\sigma_i^2c^2\sigma_{xi}^2}+const.\\
&= \frac{1}{2\rho}\sum_i [(\bar{z}-ra_i)^2+\sigma_i^2+r^2b^2\sigma_i^2+r^2c^2\sigma_{xi}^2-2rb\sigma^2]\left[\frac{b^2}{c^2\sigma_{xi}^2}+\frac{1}{\frac{1}{\alpha_{ji}}\sigma_i^2}\right]+const.\\
&= \frac{1}{2\rho}\sum_i G_i\alpha_{ji}+const.
\end{aligned}
$$

where

$$
G_i=\frac{(\bar{z}-ra_i)^2}{\sigma_i^2}+(1-rb_i)^2+r^2c^2\frac{\sigma_{xi}^2}{\sigma_i^2}
$$

So, the objective is linear in α_{ji}, subject to constraint

$$
\prod_i \alpha_{ji}\le e^{2K_1}.
$$

A.1.2 Proof of Proposition 1

First, we need to derive the shadow value of information G_i as function of fundamentals and information choices. Start with demand for asset i from investor j who choses learning α_{ji}:

$$
\begin{aligned}
q_{ji} &= \frac{\mu_{ji}-rp_i}{\rho\hat{\sigma}_{ji}^2},\\
\mu_{ji} &= s_{ji}+\frac{cov_j(z_i,p_i)}{\sigma_{pji}^2}(p_i-E_j[p_i])\\
\hat{\sigma}_{ji}^2 &= \frac{1}{\alpha_{ji}}\sigma_i^2-\frac{cov_j^2(z_i,p_i)}{\sigma_{pji}^2}
\end{aligned}
$$

where

$$
cov_j(z_i,p_i)=b_i\frac{\sigma_i^2}{\alpha_{ji}}
$$

$$
\sigma_{pji}^2=b_i^2\frac{\sigma_i^2}{\alpha_{ji}}+c_i^2\sigma_{xi}^2
$$

Plugging in for the posterior variance and mean, we get

$$
\begin{aligned}
q_{ji} &= \frac{b^2\sigma_i^2\frac{1}{\alpha_{ji}}+c_i^2\sigma_{xi}^2}{c_i^2\sigma_i^2\sigma_{xi}^2\frac{1}{\alpha_{ji}}}(\mu_{ji}-rp_i)=\frac{b^2\sigma_i^2\frac{1}{\alpha_{ji}}+c_i^2\sigma_{xi}^2}{c_i^2\sigma_i^2\sigma_{xi}^2\frac{1}{\alpha_{ji}}}(s_{ji}+\frac{b\sigma_i^2\frac{1}{\alpha_{ji}}}{b^2\sigma_i^2\frac{1}{\alpha_{ji}}+c_i^2\sigma_{xi}^2}[p_i-a_i-b_i(s_{ji}-\bar{z})]-rp_i)\\
&= \frac{1}{\rho}\left[s_{ji}\frac{\alpha_{ji}}{\sigma_i^2}+\frac{b_i}{c_i^2\sigma_{xi}^2}(p_i-a_i+b_i\bar{z})-\frac{b^2\sigma_i^2\frac{1}{\alpha_{ji}}+c_i^2\sigma_{xi}^2}{c_i^2\sigma_i^2\sigma_{xi}\frac{1}{\alpha_{ji}}}rp_i\right]
\end{aligned}
$$

Let $\hat{\lambda}_i$ be the mass of agents learning about asset i, with $\sum_i\hat{\lambda}_i=\lambda_1$. Denote the average quantity of the learning agents as $\hat{q}_i$ and the average quantity of the non-learning agents as q_i. Then, market clearing is

$$
x_i=\hat{\lambda}_i\hat{q}_i+(1-\hat{\lambda}_i)q_i
$$

The cross-sectional average of the signal is $\bar{z}+(1-\frac{1}{\alpha_i})\varepsilon_i$, where α_i is the common value of alpha that learning agents choose (due to the solution being a corner, as we show in A.1.1). Then, market clearing becomes

$$\rho x_i = \hat{\lambda}_i \left[\frac{\alpha_i}{\sigma_i^2} (\bar{z} + (1 - \frac{1}{\alpha_i})\varepsilon_i) - \frac{b^2 \sigma_i^2 \frac{1}{\alpha_{ji}} + c_i^2 \sigma_{xi}^2}{c_i^2 \sigma_i^2 \sigma_{xi}^2 \frac{1}{\alpha_{ji}}} r p_i \right] + (1 - \hat{\lambda}_i) \left[\frac{1}{\sigma_i^2} \bar{z} - \frac{b^2 \sigma_i^2 + c_i^2 \sigma_{xi}^2}{c_i^2 \sigma_i^2 \sigma_{xi}^2} r p_i \right] + \frac{b_i}{c_i^2 \sigma_{xi}^2} (p_i - a_i + b_i \bar{z})$$

Using

$$\hat{\lambda}_i \frac{b^2 \sigma_i^2 \frac{1}{\alpha_{ji}} + c_i^2 \sigma_{xi}^2}{c_i^2 \sigma_i^2 \sigma_{xi}^2 \frac{1}{\alpha_{ji}}} + (1 - \hat{\lambda}_i) \frac{b^2 \sigma_i^2 + c_i^2 \sigma_{xi}^2}{c_i^2 \sigma_i^2 \sigma_{xi}^2} = \frac{b_i^2 \sigma_i^2 + c_i^2 \sigma_{xi}^2 (\hat{\lambda}_i \alpha_i + 1 - \hat{\lambda}_i))}{c_i^2 \sigma_i^2 \sigma_{xi}^2}$$

the market clearing is

$$\rho x_i = \bar{z}[\frac{\hat{\lambda}_i \alpha_i + 1 - \hat{\lambda}_i}{\sigma_i^2} + \frac{b_i^2}{c_i^2 \sigma_{xi}^2}] + \frac{\hat{\lambda}_i (\alpha_i - 1)}{\sigma_i^2} \varepsilon_i - \frac{b_i}{c_i^2 \sigma_{xi}^2} a_i - \frac{-b_i \sigma_i^2 + r b_i^2 \sigma_i^2 + r c_i^2 \sigma_{xi}^2 (\hat{\lambda}_i \alpha_i + 1 - \hat{\lambda}_i))}{c_i^2 \sigma_i^2 \sigma_{xi}^2} p_i,$$

and therefore

$$p_i \left[\frac{r b_i^2}{c_i^2 \sigma_{xi}^2} + \frac{r(\hat{\lambda}_i \alpha_i + 1 - \hat{\lambda}_i)}{\sigma_i^2} - \frac{b_i}{c_i^2 \sigma_{xi}^2} \right] = -\rho(\bar{x}_i + \nu_i) + \bar{z}[\frac{\hat{\lambda}_i \alpha_i + 1 - \hat{\lambda}_i}{\sigma_i^2} + \frac{b_i^2}{c_i^2 \sigma_{xi}^2}] + \frac{\hat{\lambda}_i (\alpha_i - 1)}{\sigma_i^2} \varepsilon_i - \frac{b_i}{c_i^2 \sigma_{xi}^2} a_i.$$

We get

$$\frac{b_i}{c_i} = \frac{\hat{\lambda}_i (\alpha_i - 1)}{\rho \sigma_i^2},$$

and

$$\frac{1}{c_i} = \frac{1}{\rho} \left[r \frac{\hat{\lambda}_i^2 (\alpha_i - 1)^2}{\rho^2 \sigma_i^4 \sigma_{xi}^2} + r \frac{(\hat{\lambda}_i \alpha_i + 1 - \hat{\lambda}_i)}{\sigma_i^2} - \frac{\hat{\lambda}_i (\alpha_i - 1)}{\rho \sigma_i^2 \sigma_{xi}^2} \frac{1}{c_i} \right],$$

and so

$$c_i = \frac{\rho + \frac{\hat{\lambda}_i (\alpha_i - 1)}{\rho \sigma_i^2 \sigma_{xi}^2}}{r \frac{\hat{\lambda}_i^2 (\alpha_i - 1)^2}{\rho^2 \sigma_i^4 \sigma_{xi}^2} + r \frac{(\hat{\lambda}_i \alpha_i + 1 - \hat{\lambda}_i)}{\sigma_i^2}} = \frac{\rho \sigma_i}{r} \frac{\hat{\lambda}_i (\alpha_i - 1) + \rho^2 \sigma_i^2 \sigma_{xi}^2}{\hat{\lambda}_i^2 (\alpha_i - 1)^2 + (1 - \hat{\lambda}_i + \hat{\lambda}_i \alpha_i) \rho^2 \sigma_i^2 \sigma_{xi}^2}.$$

Then

$$b_i = \frac{\hat{\lambda}_i (\alpha_i - 1)}{r} \frac{\hat{\lambda}_i (\alpha_i - 1) + \rho^2 \sigma_i^2 \sigma_{xi}^2}{\hat{\lambda}_i^2 (\alpha_i - 1)^2 + (1 - \hat{\lambda}_i + \hat{\lambda}_i \alpha_i) \rho^2 \sigma_i^2 \sigma_{xi}^2},$$

so that

$$1 - r b_i = \frac{\rho^2 \sigma_i^2 \sigma_{xi}^2}{\hat{\lambda}_i^2 (\alpha_i - 1)^2 + (1 - \hat{\lambda}_i + \hat{\lambda}_i \alpha_i) \rho^2 \sigma_i^2 \sigma_{xi}^2}.$$

Finally,

$$a_i = \frac{-\rho \bar{x}_i + \bar{z}[\frac{\hat{\lambda}_i \alpha_i + 1 - \hat{\lambda}_i}{\sigma_i^2} + \frac{b_i^2}{c_i^2 \sigma_{xi}^2}] - \frac{b_i}{c_i^2 \sigma_{xi}^2} a_i}{\frac{r b_i^2}{c_i^2 \sigma_{xi}^2} + \frac{r(\hat{\lambda}_i \alpha_i + 1 - \hat{\lambda}_i)}{\sigma_i^2} - \frac{b_i}{c_i^2 \sigma_{xi}^2}},$$

$$a_i r \left[\frac{b_i^2}{c_i^2 \sigma_{xi}^2} + \frac{(\hat{\lambda}_i \alpha_i + 1 - \hat{\lambda}_i)}{\sigma_i^2} \right] = -\rho \bar{x}_i + \bar{z}[\frac{\hat{\lambda}_i \alpha_i + 1 - \hat{\lambda}_i}{\sigma_i^2} + \frac{b_i^2}{c_i^2 \sigma_{xi}^2}],$$

$$r a_i = \bar{z} - \frac{\rho \bar{x}_i}{\frac{b_i^2}{c_i^2 \sigma_{xi}^2} + \frac{(\hat{\lambda}_i \alpha_i + 1 - \hat{\lambda}_i)}{\sigma_i^2}},$$

and therefore, we have

$$\bar{z} - r a_i = \frac{\rho^3 \sigma_i^4 \sigma_{xi}^2 \bar{x}_i}{\hat{\lambda}_i^2 (\alpha_i - 1)^2 + (1 - \hat{\lambda}_i + \hat{\lambda}_i \alpha_i) \rho^2 \sigma_i^2 \sigma_{xi}^2}.$$

Given the price coefficients, we can express G_i as a function of the masses of agents learning about asset i:

$$G_i = \rho^2\sigma_i^2\sigma_{xi}^2 \frac{\rho^2\sigma_i^2\sigma_{xi}^2 + \rho^4\sigma_i^4\sigma_{xi}^2\bar{x}_i^2 + (\hat{\lambda}_i(\alpha_i - 1) + \rho^2\sigma_i^2\sigma_{xi}^2)^2}{[\hat{\lambda}_i^2(\alpha_i - 1)^2 + (1 - \hat{\lambda}_i + \hat{\lambda}_i\alpha_i)\rho^2\sigma_i^2\sigma_{xi}^2]^2}.$$

To see **Part 1** of the proposition, note that obviously, G_i is increasing in $\bar{x}_i$. Second, it is increasing in σ_{xi}^2. The partial derivative is

$$\frac{\partial G_i}{\partial \sigma_{xi}^2} =$$

$$\rho^2\sigma_i^2 \frac{\hat{\lambda}_i^4(\alpha_i - 1)^4 + 3\hat{\lambda}_i^3(\alpha_i - 1)^3\rho^2\sigma_i^2\sigma_{xi}^2 + \rho^6\sigma_i^6\sigma_{xi}^6 + \hat{\lambda}_i(\alpha_i - 1)\rho^6\sigma_i^6\sigma_{xi}^6 + \hat{\lambda}_i^2(\alpha_i - 1)^2\rho^2\sigma_i^2\sigma_{xi}^2(1 + \rho^2\sigma_i^2(3\sigma_{xi}^2 + 2\bar{x}_i^2))}{[\hat{\lambda}_i^2(\alpha_i - 1)^2 + (1 - \hat{\lambda}_i + \hat{\lambda}_i\alpha_i)\rho^2\sigma_i^2\sigma_{xi}^2]^3} > 0$$

Finally, the partial derivative with respect to asset's fundamental volatility is

$$\frac{\partial G_i}{\partial \sigma_i^2} = \frac{\rho^2\sigma_{xi}^2}{((-1 + \alpha_i)^2\hat{\lambda}_i^2 + (1 + (-1 + \alpha_i)\hat{\lambda}_i)\rho^2\sigma_i^2\sigma_{xi}^2)^3} \times$$

$$[(\alpha_i - 1)^4\hat{\lambda}_i^4 + 3(\alpha_i - 1)^3\hat{\lambda}_i^3\rho^2\sigma_i^2\sigma_{xi}^2 + \rho^6\sigma_i^6\sigma_{xi}^4(\sigma_{xi}^2 + \bar{x}_i^2) + (\alpha_i - 1)\hat{\lambda}_i\rho^6\sigma_i^6\sigma_{xi}^4(\sigma_{xi}^2 + \bar{x}_i^2)$$

$$+(\alpha_i - 1)^2\hat{\lambda}_i^2\rho^2\sigma_i^2\sigma_{xi}^2(1 + 3\rho^2\sigma_i^2(\sigma_{xi}^2 + \bar{x}_i^2))] > 0$$

That means that the investors, ceteris paribus, have preferences towards asset with high and noisy supply, and high volatility of returns.

For **Part 2**, it is enough to show that the shadow value of learning about asset i are decreasing in the mass of agents learning about that asset, that is, $\frac{\partial G_i}{\partial \hat{\lambda}} < 0$:

$$\frac{\partial G_i}{\partial \hat{\lambda}} = -2(\alpha_i - 1)\rho^2\sigma_i^2\sigma_{xi}^2 \times$$

$$\frac{((\alpha_i - 1)^3\hat{\lambda}_i^3 + 3(\alpha_i - 1)^2\hat{\lambda}_i^2\rho^2\sigma_i^2\sigma_{xi}^2 + \rho^6\sigma_i^6\sigma_{xi}^4(\sigma_{xi}^2 + \bar{x}^2) + (\alpha_i - 1)\hat{\lambda}_i\rho^2\sigma_i^2\sigma_{xi}^2(1 + \rho^2\sigma_i^2(3\sigma_{xi}^2 + 2\bar{x}^2)))}{[\hat{\lambda}_i^2(\alpha_i - 1)^2 + (1 - \hat{\lambda}_i + \hat{\lambda}_i\alpha_i)\rho^2\sigma_i^2\sigma_{xi}^2]^3} < 0.$$

A.2 Monopoly

Our second analytical case is that of a single large informed investor ($l = 1$), and no free price learning by the fringe investors. In this case, the price impact term simplifies to $\frac{dp_i}{dq_{ji}} = \frac{\rho\sigma_i^2\lambda_1}{r\lambda_0}$ and the β terms are $\beta_{0ji} = 0$, and $\beta_{1ji} = \beta_{2ji} \equiv \beta_i$, as there is no additional information in the price over and above that coming from the private signal of the monopolist (and hence $\gamma_{1i} = 0$). The monopolist's expected utility is given by:

$$U_1 = \frac{1}{2\rho}\sum_{i=1}^{n} \frac{\frac{L_i}{\lambda_0}\alpha_i + \lambda_0(\alpha_i - 1)}{\lambda_0 + 2\lambda_1\alpha_i}. \tag{18}$$

We simplify the learning notation by dropping the index on α_i and we summarize the learning benefit as $L_i \equiv \rho^2(\bar{x}_i^2 + \sigma_{xi}^2)\sigma_i^2$. The marginal utility from increasing learning about asset i is:

$$\frac{\partial U}{\partial \alpha_i} = \frac{L_i + 1 - \lambda_1^2}{(\lambda_0 + 2\lambda_1\alpha_i)^2}.$$

Optimality dictates that, for all i:

$$\frac{L_i + 1 - \lambda_1^2}{(\lambda_0 + 2\lambda_1\alpha_i)^2}\alpha_i \leq \theta \tag{19}$$

where θ is the Lagrange multiplier on the capacity constraint $\sum_{i=1}^{n}\log(\alpha_i) = 2K$. The condition holds with equality whenever $\alpha_i > 1$.

Define the gain from learning about asset i as $M_i \equiv \frac{L_i + 1 - \lambda_1^2}{(\lambda_0 + 2\lambda_1\alpha_i)^2}\alpha_i$. M_i is increasing in L_i, and hence, just like the competitive investor, the monopolist has preference for learning about assets that are volatile (σ_i^2 or σ_{xi}^2) or are in large supply ($\bar{x}_i$). However, the monopolist has an incentive to learn about multiple assets, as long as his size and information capacity are large enough. Specifically, without loss of generality, let the assets be sorted, such that $L_1 > L_2 > ... > L_n$. Then, the following holds:

Proposition 2. *(i) For every K, there exists $\underline{\lambda} > 0$, such that for all $\lambda_1 < \underline{\lambda}$, the monopolist learns only about one asset, $i = 1$.*
(ii) Let $\lambda_1 > 1/3$. Then M_i is strictly decreasing in α_i for all i, and there exists $\bar{K}$ such that, for $K > \bar{K}$, there exists a set of cutoffs $\lambda_1(N)$ increasing in N, such that for $\lambda_1 > \lambda_1(N)$, the monopolist learns about at least N assets with the highest L_is. Moreover, $\lambda_1(n) < 1$, that is, the monopolist eventually learns about all assets.

The proposition above states that for λ_1 large enough and if capacity is not too small, as the monopolist grows in size, his learning choice involves a growing number of assets, and eventually the monopolist learns about all assets. It is important to note that even though, from the learning perspective, the investor in (i) behaves like a competitive investor, the aggregate allocation is different and here, actually $PI_i > 0$ only for asset 1.

Implications for price informativeness Price informativeness in the monopoly case is:

$$PI_i = \frac{cov(p_i, z_i)}{\sigma_{pi}} = \frac{\sigma_i \lambda_1 \beta_i \frac{\alpha_i - 1}{\alpha_i}}{\sqrt{\frac{\sigma_{xi}^2}{\sigma_i^2} + \left[\lambda_1 \beta_i \frac{\alpha_i - 1}{\alpha_i}\right]^2 + \lambda_1^2 \beta_i^2 \frac{\alpha_i - 1}{\alpha_i^2}}},$$

where the information pass-through term, $\lambda_1 \beta_i$, is

$$\lambda_1 \beta_i = \frac{1}{\rho \sigma_i^2} \frac{\lambda_1 (1 - \lambda_1) \alpha_i}{1 - \lambda_1 + \lambda_1 \alpha_i}.$$

In equilibrium, the information pass-through is always non-negative, but non-monotonic, increasing for small values of λ_1, then eventually decreasing. The following proposition summarizes this result:

Proposition 3. *There exist λ_L and λ_H such that information pass-through is strictly increasing in λ_1 for $\lambda_1 < \lambda_L$, and strictly decreasing for $\lambda_1 > \lambda_H$. It is always non-negative and equal to zero for the extreme values of $\lambda_1 \in \{0, 1\}$.*

The implication of Proposition 3 is that the non-monotonicity of the information pass-through is going to contribute to the potential non-monotonicity of price informativeness on an asset-by-asset basis, and quantitatively it can contribute to the non-monotonicity of the average price informativeness. Propositions 2 and 3 imply that for small λ_1, information pass-through is the only determinant of the shape of price informativeness, while Proposition 2 implies that information pass-through is the main determinant of the shape of price informativeness as λ_1 approaches 1. In fact, price informativeness converges to zero in that case, while learning still remains positive. That is because as λ_1 approaches 1, information pass-through approaches 0. Corollary 2 formalizes this result. This is in contrast to the perfectly competitive result of Corollary 1, where we show that PI is monotonic in the size of the informed investor sector.

Corollary 2. *Price informativeness is strictly increasing in λ_1 for $\lambda_1 < \lambda_L$, and strictly decreasing in λ_1 for $\lambda_1 > \lambda_H$. It is non-negative for all λ_1 and zero for $\lambda_1 \in \{0, 1\}$.*

In equilibrium, the remaining factor that affects price informativeness is the equilibrium adjustment of learning α_i, which can also be non-monotonic. The relative contribution of these two channels outside the $[0, \lambda_L] \cup [\lambda_H, 1]$ set depends on the specifics of the parameters of the model.

A.2.1 Derivation of the Monopolist's Utility (18)

Under the monopoly structure and no learning from prices, we have: $\gamma_{ji} = 0$ and also $\beta_1 = \beta_2$ and $dp/dq = \rho\sigma_i^2\lambda_1/(r\lambda_0)$, such that

$$\frac{r}{\rho}\frac{dp}{dq} = \sigma_i^2\frac{\lambda_1}{\lambda_0} \equiv \sigma_i^2\hat{\lambda},$$

where $\hat{\lambda} \equiv \frac{\lambda_1}{\lambda_0}$. The ex-ante information decision maximizes:

$$E_0U_j = \frac{1}{2\rho}\sum_{i=1}^{n} E_0(\hat{\mu}_{ji} - rp_i)^2 \frac{\hat{\sigma}_{ji}^2 + 2r\frac{dp_i}{\rho dq_{ji}}}{(\hat{\sigma}_{ji}^2 + r\frac{dp_i}{\rho dq_{ji}})^2},$$

which becomes

$$E_0U_j = \frac{1}{2\rho}\sum_{i=1}^{n} E_0(\hat{\mu}_{ji} - rp_i)^2 \frac{\hat{\sigma}_{ji}^2 + 2\sigma_i^2\hat{\lambda}}{(\hat{\sigma}_{ji}^2 + \sigma_i^2\hat{\lambda})^2}.$$

We will use

$$rp_i = -\frac{1}{\Delta_i}x_i\frac{\rho\sigma_i^2}{\lambda_0} + \frac{1}{\Delta_i}\frac{\rho\sigma_i^2}{\lambda_0}\sum_{j=1}^{l}\lambda_j\beta_{0ji} + \frac{1}{\Delta_i}\frac{\rho\sigma_i^2}{\lambda_0}\sum_{j=1}^{l}\lambda_j\beta_{1ji}s_{ji} + \frac{1}{\Delta_i}\bar{z},$$

where

$$\Delta_i = \left(1 + \frac{\rho\sigma_i^2}{\lambda_0}\sum_{j=1}^{l}\lambda_j\beta_{2ji}\right).$$

As before, we compute $E_{0j}(\widehat{\mu}_{ji} - rp_i)^2 = \widehat{R}_i^2 + \widehat{V}_{ji}$, where $\widehat{R}_i$ and $\widehat{V}_{ji}$ denote the ex-ante mean and variance of expected excess returns,

$$\hat{R}_i = E_{ji0}(\hat{\mu}_{ji} - rp_i) = \bar{z}_i + \frac{1}{\Delta_i}\bar{x}_i\frac{\rho\sigma_i^2}{\lambda_0} - \frac{1}{\Delta_i}\frac{\rho\sigma_i^2}{\lambda_0}\sum_{j=1}^{l}\lambda_j\beta_{0ji} - \frac{1}{\Delta_i}\frac{\rho\sigma_i^2}{\lambda_0}\sum_{j=1}^{l}\lambda_j\beta_{1ji}\bar{z}_i - \frac{1}{\Delta_i}\bar{z}_i =$$

$$= \frac{1}{\Delta_i}\bar{x}_i\frac{\rho\sigma_i^2}{\lambda_0} - \frac{1}{\Delta_i}\frac{\rho\sigma_i^2}{\lambda_0}\sum_{j=1}^{l}\lambda_j\beta_{0ji} + \bar{z}_i(1 - \frac{1 + \frac{\rho\sigma_i^2}{\lambda_0}\sum_{j=1}^{l}\lambda_j\beta_{1ji}}{\Delta_i})$$

$$= \frac{1}{\Delta_i}\bar{x}_i\frac{\rho\sigma_i^2}{\lambda_0}$$

We now compute

$$\hat{V}_{ji} = var_0(\mu - rp_i)$$

$$var_0(\hat{\mu}_{ji} - rp_i) = var_0(\hat{\mu}_{ji}) + var_0(rp_i) - 2rcov(\mu_{ji}, p_i)$$

We have

$$\begin{aligned}
var_0(\mu_{ji}) &= var_0(s_{ji} + \gamma_{ji}(p_i - E_j[p_i]) = var_0(s_{ji}) + \gamma_{ji}^2 var(p_i) - 2\gamma_{ji}cov(s_{ji}, p_i) = \\
&= (1 - 1/\alpha_{ji})\sigma_i^2 \\
var_0(rp_i) &= \left(\frac{1}{\Delta_i}\frac{\rho\sigma_i^2}{\lambda_0}\right)^2\left(\sigma_{ix}^2 + \lambda_1^2\beta_i^2(1 - 1/\alpha_{ji})\sigma_i^2\right) \\
2rcov(\hat{\mu}_{ji}, p_i) &= 2rcov(s_{ji} + \gamma_{ji}(p_i - E_j[p_i]), p_i) = 2rcov(s_{ji}, p_i) = \\
&= 2\frac{\rho\sigma_i^2}{\lambda_0\Delta_i}\lambda_j\beta_i(1 - 1/\alpha_{ji})\sigma_i^2
\end{aligned}$$

Summing up:

$$\begin{aligned}\hat{V}_i &= (1-1/\alpha_{ji})\sigma_i^2(1+\left(\frac{1}{\Delta_i}\frac{\rho\sigma_i^2}{\lambda_0}\right)^2(\frac{\sigma_{xi}^2}{(1-1/\alpha_i)\sigma_i^2}+\lambda_1^2\beta_i^2)-2\frac{1}{\Delta_i}\frac{\rho\sigma_i^2}{\lambda_0}\lambda_1\beta_i) \\ &= (1-1/\alpha_{ji})\sigma_i^2[\frac{1}{\Delta_i^2}+(\frac{1}{\Delta_i}\frac{\rho\sigma_i^2}{\lambda_0})^2\frac{\sigma_{xi}^2}{(1-1/\alpha_i)\sigma_i^2}] \end{aligned} \tag{20}$$

So, $R_i^2 + V_i$ is:

$$\begin{aligned}&(\frac{1}{\Delta_i}\bar{x}_i\frac{\rho\sigma_i^2}{\lambda_0})^2+(\sigma_i^2-\hat{\sigma}_i^2)[\frac{1}{\Delta_i^2}+\frac{(\frac{1}{\Delta_i}\frac{\rho\sigma_i^2}{\lambda_0})^2\sigma_{xi}^2}{(\sigma_i^2-\hat{\sigma}_i^2)}] \\ &= \frac{1}{\Delta_i^2}[\frac{\rho^2\sigma_i^4}{\lambda_0^2}(\bar{x}_i^2+\sigma_{xi}^2)+(\sigma_i^2-\hat{\sigma}_i^2)]\end{aligned}$$

Now,

$$\Delta_i = 1+\frac{\rho\sigma_i^2}{\lambda_0}\beta_i\lambda_1, \quad rdp/dq = \rho\sigma_i^2\lambda_1/(\lambda_0) \text{ and } \beta = \frac{1}{\rho\hat{\sigma}_i^2 + rdp/dq},$$

so that

$$\Delta = \frac{\hat{\sigma}_i^2+2\sigma_i^2\lambda_1/\lambda_0}{\hat{\sigma}_i^2+\sigma_i^2\lambda_1/\lambda_0}.$$

Given that, the utility is

$$\begin{aligned}E_0U_j &= \frac{1}{2\rho}\sum_{i=1}^n\frac{(\hat{\sigma}_i^2+\sigma_i^2\lambda_1/\lambda_0)^2}{(\hat{\sigma}_i^2+2\sigma_i^2\lambda_1/\lambda_0)^2}[\frac{\rho^2\sigma_i^4}{\lambda_0^2}(\bar{x}_i^2+\sigma_{xi}^2)+(\sigma_i^2-\hat{\sigma}_i^2)]\frac{\hat{\sigma}_{ji}^2+2\sigma_i^2\hat{\lambda}}{(\hat{\sigma}_{ji}^2+\sigma_i^2\hat{\lambda})^2} \\ &= \frac{1}{2\rho}\sum_{i=1}^n[\frac{\rho^2\sigma_i^4}{\lambda_0^2}(\bar{x}_i^2+\sigma_{xi}^2)+(\sigma_i^2-\hat{\sigma}_i^2)]\frac{1}{(\hat{\sigma}_i^2+2\sigma_i^2\hat{\lambda})},\end{aligned}$$

which gives (18)

$$= \frac{1}{2\rho}\sum_{i=1}^n[\frac{\rho^2\sigma_i^2}{\lambda_0^2}(\bar{x}_i^2+\sigma_{xi}^2)\alpha_i+\alpha_i-1]\frac{1}{\lambda_0+2\lambda_1\alpha_i}.$$

A.2.2 Proof of Proposition 2

Proof. (i) $\frac{\partial M_i}{\partial \alpha_i} = \frac{(L_i+1-\lambda_1^2)(\lambda_0+2\lambda_1\alpha_i)}{(\lambda_0+2\lambda_1\alpha_i)^3} - \alpha_i\frac{(L_i+1-\lambda_1^2)4\lambda_1}{(\lambda_0+2\lambda_1\alpha_i)^3}$, which is equal in sign to

$$1-\lambda_1-2\lambda_1\alpha_i.$$

Hence, given K, for λ_1 such that

$$\lambda_1 < \frac{1}{1+2e^{2K}} \equiv \underline{\lambda},$$

M_i is strictly increasing for all i. The M_is also always preserve the rank implied by L_i and therefore, the monopolist will learn only about the asset with the highest L_i.

(ii) By the analysis in (i), for $\lambda_1 > 1/3$, M_i is strictly decreasing in α_{1i} for all i. Moreover, for any two assets that are learned about, i and k, it has to be true that

$$\frac{L_i+1-\lambda_1^2}{(1-\lambda_1+2\lambda_1\alpha_i)^2}\alpha_i = \frac{L_k+1-\lambda_1^2}{(1-\lambda_1+2\lambda_1\alpha_k)^2}\alpha_k$$

Define the common value, which depends on λ_1 and K, as $\bar{M}(\lambda_1, K)$. Suppose that the monopolist learns about $N < n$ assets. We know that

$$\lim_{\lambda_1\to 1}\bar{M}(\lambda_1,K) \equiv \bar{M} = \frac{L_i}{4\alpha_i} = \frac{L_k}{4\alpha_k} \tag{21}$$

and in that case

$$N\log(\bar{M}) = \sum_{i=1}^{N}\log(L_i) - \sum_{i=1}^{N}\log(\alpha_i) = \sum_{i=1}^{N}\log L_i - 2K.$$

Suppose that asset j is never learned about. Then it must be true that for all λ_1, K, $\bar{M}(\lambda_1, K) > \frac{L_j+1-\lambda_1^2}{(1+\lambda_1)^2}$. That condition is violated for any asset with positive L_j and $K > \frac{1}{2}\sum_{i=1}^{n}\log(L_i)$. This defines $\bar{K} = \frac{1}{2}\sum_{i=1}^{n}\log(L_i)$. Additionally,

$$\frac{d\bar{M}(\lambda_1, K)}{d\lambda_1} = \frac{\partial\bar{M}(\lambda_1, K)}{\partial\lambda_1} + \frac{\partial\bar{M}(\lambda_1, K)}{\partial\alpha_i}\frac{d\alpha_i}{d\lambda_1} < 0. \tag{22}$$

To see this, note first that the two partial derivatives are negative. The term $d\alpha_i/d\lambda_1$ can be positive or negative, depending on the asset. However, even if it is negative, its effect cannot in equilibrium change the sign of (22). If it did, that is, if $\bar{M}(\lambda_1, K)$ increased after an increase in λ_1, that would mean that M_i went up for all actively traded assets, and that can only be achieved if α_{1i} goes down for all actively traded assets, clearly violating optimality. This argument does not rely on the number of assets that are learned about being constant.

Equation (22) implies that assets are learned about according to ranking by L_i. As λ_1 increases the monopolist learns first about the highest-L_i asset, then the second highest L_i, etc. until all assets are learned about. A sufficient condition for that is $K > \bar{K}$ and positive L_is. Then all assets are learned about as λ_1 approaches 1, but is still bounded away from 1. The specific $\lambda_1(N)$ are defined by:

$$\frac{L_{N-1}+1-\lambda_1^2}{(1-\lambda_1+2\lambda_1\alpha_{1(N-1)})^2}\alpha_{1(N-1)} = \frac{L_N+1-\lambda_1^2}{(1+\lambda_1)^2}$$

which exists by monotonicity of $\bar{M}(\lambda_1, \bar{K})$ and (21).

□

A.2.3 Proof of Proposition 3

Proof. For any $\lambda_1 < \underline{\lambda}$, where $\underline{\lambda}$ is defined in Proposition 2, $\frac{d\alpha_{1i}}{d\lambda_1} = 0$ (it is e^{2K} for asset $i = 1$ and 1 otherwise. In such case,

$$\frac{d\lambda_1\beta_i}{d\lambda_1} = \frac{\partial\lambda_1\beta_i}{\partial\lambda_1} = \frac{1-2\lambda_1-\lambda_1^2\alpha_i}{(1-\lambda_1+\lambda_1\alpha_i)^2}$$

Evaluating, we have that $\frac{d\lambda_1\beta_i}{d\lambda_1} > 0$ for $\lambda_1 < \frac{\sqrt{\alpha+1}-1}{\alpha}$ (which is strictly less than 1 and strictly more than 0). We have that information pass-through is strictly increasing for $\lambda_1 < \lambda_L \equiv \min\{\underline{\lambda}, \frac{\sqrt{e^{2K}+1}-1}{e^{2K}}\}$.
For larger $\lambda_1 (thatis, > \sqrt{2}-1)$, we have:

$$\frac{d\lambda_1\beta_i}{d\lambda_1} = \frac{\partial\lambda_1\beta_i}{\partial\lambda_1} + \frac{\partial\lambda_1\beta_i}{\partial\alpha_i}\frac{d\alpha_i}{d\lambda_1},$$

where

$$\frac{\partial\lambda_1\beta_i}{\partial\lambda_1} = \frac{1-2\lambda_1-\lambda_1^2\alpha_i}{(1-\lambda_1+\lambda_1\alpha_i)^2} < 0,$$

$$\frac{\partial\lambda_1\beta_i}{\partial\alpha_i} = \frac{\lambda_1(1-\lambda_1)^2}{(1-\lambda_1+\lambda_1\alpha_i)^2} \geq 0.$$

Additionally,

$$\lim_{\lambda_1\to 1}\frac{\partial\lambda_1\beta_i}{\partial\lambda_1} = -\frac{1+\alpha_i}{\alpha_i^2} < -\frac{1+e^{2K}}{e^{4K}},$$

and so it is bounded away from 0, and

$$\lim_{\lambda_1\to 1}\frac{\partial\lambda_1\beta_i}{\partial\alpha_i} = 0.$$

By arguments underlying the proof of Proposition 2, $d\alpha_i/d\lambda_1$ is bounded from above and below. If it were not bounded from below, then it would have to be the case that $\frac{d\bar{M}(\lambda_1,K)}{d\lambda_1}$ in (22) is positive for some i and some value of λ_1, which creates a contradiction as it would mean that for that value of λ_1, the capacity constraint is slack, violating optimality. The immediate consequence is that it is also bounded from above as otherwise the capacity constraint $\sum_i \log(\alpha_i) \leq 2K$ would be violated for some λ_1. Based on these results, we can conclude that there exists a cutoff value $\lambda_H < 1$ such that for all $\lambda_1 > \lambda_H$, we have $\frac{d\lambda_1\beta_i}{d\lambda_1} < 0$. The non-negativity and zero values at points $\lambda_1 \in \{0, 1\}$ follow trivially from the equation. □

A.2.4 Proof of Corollary 2

Proof. We have

$$\frac{dPI}{d\lambda_1} = \frac{\partial PI}{\partial\alpha_i}\frac{d\alpha_i}{d\lambda_1} + \frac{\partial PI}{\partial(\lambda_1\beta_i)}\frac{d\lambda_1\beta_i}{d\lambda_1},$$

where

$$\frac{\partial PI}{\partial\alpha_i} = \frac{\lambda_1\beta_i\frac{\sigma_{xi}^2}{\sigma_i^2\alpha_i^2} + (\lambda_1\beta_i)^3\frac{\alpha_i^2-2\alpha_i+1}{2\alpha_i^4}}{\left(\sqrt{\frac{\sigma_{xi}^2}{\sigma_i^2} + \left[\lambda_1\beta_i\frac{\alpha_i-1}{\alpha_i}\right]^2 + \lambda_1^2\beta_i^2\frac{\alpha_i-1}{\alpha_i^2}}\right)^3} > 0$$

and

$$\frac{\partial PI}{\partial\lambda_1\beta_i} = \frac{\alpha_i-1}{\alpha_i}\left[\frac{1}{2}\frac{1}{\sqrt{\frac{\sigma_{xi}^2}{\sigma_i^2} + \left[\lambda_1\beta_i\frac{\alpha_i-1}{\alpha_i}\right]^2 + \lambda_1^2\beta_i^2\frac{\alpha_i-1}{\alpha_i^2}}} + 2\frac{\sigma_{xi}^2}{\sigma_i^2(\lambda_1\beta_i)^3}\right] > 0.$$

Additionally, for $\lambda_1 < \lambda_L$, we have that $d\alpha_i/d\lambda_1 = 0$ and by Proposition 3, $\frac{d\lambda_1\beta_i}{d\lambda_1} > 0$, and so $dPI/d\lambda_1 > 0$. For large λ_1, we have

$$\lim_{\lambda_1\to 1}\frac{\partial PI_i}{\partial\alpha_i} = 0.$$

It follows that, with a bounded derivative $d\alpha_i/d\lambda_1$, $\frac{\partial PI}{\partial\lambda_1\beta_i} > 0$, and $\frac{d\lambda_1\beta_i}{d\lambda_1} < 0$ bounded away from zero, we have that $dPI_i/d\lambda_1 < 0$ for λ_1 sufficiently close to 1, but not necessarily equal to 1. □

A.3 Duopoly

We now turn to a setting with two large investors, j and k. Specifically, we study the impact of the investors' sizes, λ_j and λ_k, with $\lambda_j + \lambda_k = 1 - \lambda_0$, on the optimal choices of demand schedules, summarized by β_{1ji} and β_{1ki}, in a special case with exogenous information choices, α_{ji}, α_{ki}, and no learning from prices. Under these assumptions, $\beta_{2ji} = \beta_{1ji}$ and $\beta_{0ji} = 0$; henceforth, we use the notation β_{ji} for any non-zero β.[45] Despite the additional assumptions, it is useful to characterize the choice of demand schedules and its relation to information pass-through and price informativeness, as it helps us understand the additional role of asymmetric size and arbitrary information structures in a standard model with exogenous information (e.g., Kyle (1989)). In particular, we can isolate the effects of a passive investor's growing size on an active investor's information pass-through.

Our first result, summarized in Lemma 1, shows that, holding information and the oligopolists' total size constant, an oligopolist's own β_{ji} is negatively related to his own size and positively related to the other oligopolist's size, thus creating a force for lower information pass-through ($\omega_{ji} \equiv \lambda_j\beta_{ji}$) for larger oligopolists. The overall effect on information pass-through is ambiguous and depends on the size of the entire oligopoly sector $\sum_j \lambda_j$ relative to the fringe, but the negative relation between β_{ji}s and individual sizes is the only force responsible for a hump-shaped information pass-through, just like in the monopoly case of the previous section.

Lemma 1. $\frac{d\beta_{ji}}{d\lambda_j} < 0$ *and* $\frac{d\beta_{ji}}{d\lambda_k} > 0$.

[45] For the derivation of the solution to β_{ji}s, see Section A.3.1.

Next, we show that the optimal size distribution of investors is a function of their individual learning decisions. Specifically, if learning is symmetric, then the optimal size distribution is also symmetric. If learning is asymmetric, the optimal size distribution is also asymmetric. Finally, if one agent is informed, and the other agent is not, it is not necessarily optimal, from the price informativeness perspective, to make the uninformed agent atomistic. These results constitute the essence of Proposition 4.

Proposition 4. *Define* $\lambda_1^*, \lambda_2^* = argmax_{\lambda_1,\lambda_2} PI_i$ *for given* α_{1i} *and* α_{2i}*. Then:*

1. *If* $\alpha_{1i} = \alpha_{2i}$*, then* $\lambda_1^* = \lambda_2^*$
2. *If* $\alpha_{1i} \neq \alpha_{2i}$*, then* $\lambda_1^* \neq \lambda_2^*$
3. *If* $\alpha_{1i} = 1$*, then* $0 < \lambda_2^* \leq 1 - \lambda_0$

The first two results are important because of their implications for the broad theoretical literature on the topic. Previous studies modeling investment decisions of multiple informed oligopolists have dealt with the complexity of the model by assuming symmetry in sizes and information quality of those agents. Proposition 4 shows that such an assumption also implies that price informativeness is maximized at that point. However, in general, the optimal ownership structure depends on the (not necessarily symmetric) investors' information choices. The last result of the proposition may be counterintuitive—one might expect that if one agent is informed and the other is uninformed, then naturally the informed agent should be as large as possible, while the uninformed should be as small as possible. However, this intuition ignores general equilibrium forces. Recall that in the monopoly setting, if the monopolist were as large as possible (the whole market), he would not trade, implying zero price informativeness. Similarly here, making the informed agent larger could hurt price informativeness by causing his information pass-through to decrease.

The role of passive investors We can use the duopoly case to illustrate the effects of a growing size of passive investors on information revelation. We show that the information pass-through of a large active investor is always *strictly lower* if he faces a large passive competitor, rather than another large active investor. This finding illustrates an additional amplification of the negative effect of a passive investor's growing size on price informativeness, *ceteris paribus*, and directly follows from Proposition 5 below. Intuitively, lower information quantity of the large competitor increases the price impact of the oligopolist and hence reduces the benefit of acting on his private information. Of course, in equilibrium, such a response would also trigger an endogenous response of other oligopolists' learning decisions. Hence, we study the precise magnitude of these effects numerically in Section 4.

Proposition 5. $\frac{d\beta_{ji}\lambda_j}{d\hat{\sigma}_{ki}^2} < 0$*. If one duopolist exogenously experiences a reduction in his information quality, then another duopolist's information pass-through is reduced.*

A.3.1 Derivation of βs in the Duopoly Case

In this case the betas are:

$$\begin{aligned}
\beta_{0ji} &= 0 \\
\beta_{1ji} &= \frac{1}{\rho\hat{\sigma}_{ji}^2 + r\frac{dp_i}{dq_{ji}}}, \\
\beta_{2ji} &= \frac{1}{\rho\hat{\sigma}_{ji}^2 + r\frac{dp_i}{dq_{ji}}}, \\
\frac{dp_i}{dq_{ji}} &= \frac{\rho\sigma_i^2\lambda_j}{\lambda_0 r + \rho\sigma_i^2\lambda_k r\beta_{2ki}}.
\end{aligned}$$

so it is enough to find $\beta_{2ji} = \beta_{1ji}$. Plugging in:

$$\beta_{2ji} = \frac{1}{\rho\hat{\sigma}_{ji}^2 + \frac{\rho\sigma_i^2\lambda_j}{\lambda_0 + \rho\sigma_i^2\lambda_k\beta_{2ki}}}$$

which gives

$$\beta_{2ji}(\rho\hat{\sigma}^2_{ji}\lambda_0 + \rho\sigma^2_i\lambda_j) - \lambda_0 = \rho\sigma^2_i\lambda_k\beta_{2ki} - \beta_{2ji}\rho\hat{\sigma}^2_{ji}\rho\sigma^2_i\lambda_k\beta_{2ki}$$

and hence:

$$\frac{\beta_{2ji}(\rho\hat{\sigma}^2_{ji}\lambda_0 + \rho\sigma^2_i\lambda_j) - \lambda_0}{\lambda_k(\rho\sigma^2_i - \beta_{2ji}\rho\hat{\sigma}^2_{ji}\rho\sigma^2_i)} = \beta_{2ki}$$

For an oligopolist k, analogous equations give:

$$\beta_{2ki}(\rho\hat{\sigma}^2_{ki}\lambda_0 + \rho\sigma^2_i\lambda_k) - \lambda_0 = \rho\sigma^2_i\lambda_j\beta_{2ji} - \beta_{2ki}\rho\hat{\sigma}^2_{ki}\rho\sigma^2_i\lambda_j\beta_{2ji}$$

plugging in

$$\frac{\beta_{2ji}(\rho\hat{\sigma}^2_{ji}\lambda_0 + \rho\sigma^2_i\lambda_j) - \lambda_0}{\lambda_k(\rho\sigma^2_i - \beta_{2ji}\rho\hat{\sigma}^2_{ji}\rho\sigma^2_i)}(\rho\hat{\sigma}^2_{ki}\lambda_0 + \rho\sigma^2_i\lambda_k) - \lambda_0 =$$
$$\rho\sigma^2_i\lambda_j\beta_{2ji} - \frac{\beta_{2ji}(\rho\hat{\sigma}^2_{ji}\lambda_0 + \rho\sigma^2_i\lambda_j) - \lambda_0}{\lambda_k(\rho\sigma^2_i - \beta_{2ji}\rho\hat{\sigma}^2_{ji}\rho\sigma^2_i)}\rho\hat{\sigma}^2_{ki}\rho\sigma^2_i\lambda_j\beta_{2ji}$$

which is a quadratic equation:

$$\begin{aligned}
\beta^2_{2ji}A + \beta_{2ji}B + C &= 0\\
A &= \lambda_k\lambda_j\rho\sigma^2_i\rho\hat{\sigma}^2_{ji}\rho\sigma^2_i + \rho\hat{\sigma}^2_{ki}\rho\sigma^2_i\lambda_j(\rho\hat{\sigma}^2_{ji}\lambda_0 + \rho\sigma^2_i\lambda_j)\\
B &= (\rho\hat{\sigma}^2_{ji}\lambda_0 + \rho\sigma^2_i\lambda_j)(\rho\hat{\sigma}^2_{ki}\lambda_0 + \rho\sigma^2_i\lambda_k) + \lambda_k\lambda_0\rho\hat{\sigma}^2_{ji}\rho\sigma^2_i - \lambda_k\rho\sigma^2_i\rho\sigma^2_i\lambda_j\\
&\quad -\rho\hat{\sigma}^2_{ki}\rho\sigma^2_i\lambda_j\lambda_0\\
C &= -\lambda_0(\lambda_k\rho\sigma^2_i + \rho\hat{\sigma}^2_{ki}\lambda_0 + \rho\sigma^2_i\lambda_k)
\end{aligned}$$

There are two solutions for β_{2ji} and their product is negative; hence, only one positive solution—β_{2ji} is unique, and so is β_{2ki}. Collecting terms and simplifying gives:

$$\begin{aligned}
\beta^2_{2ji}A + \beta_{2ji}B + C &= 0\\
A &= \lambda_k\lambda_j\rho^3\sigma^4_i\hat{\sigma}^2_{ji} + \rho^3\hat{\sigma}^2_{ki}\sigma^2_i\lambda_j(\hat{\sigma}^2_{ji}\lambda_0 + \sigma^2_i\lambda_j)\\
B &= \rho^2\hat{\sigma}^2_{ji}\hat{\sigma}^2_{ki}\lambda^2_0 + 2\rho^2\hat{\sigma}^2_{ji}\sigma^2_i\lambda_0\lambda_k = \rho^2\lambda_0\hat{\sigma}^2_{ji}(\hat{\sigma}^2_{ki}\lambda_0 + 2\sigma^2_i\lambda_k)\\
C &= -\rho\lambda_0(\hat{\sigma}^2_{ki}\lambda_0 + 2\sigma^2_i\lambda_k)
\end{aligned}$$

Or equivalently:

$$\beta^2_{2ji}A + \beta_{2ji}B + C = 0 \tag{23}$$
$$A = \left[\lambda_k\lambda_j\rho^3\sigma^4_i\hat{\sigma}^2_{ji} + \rho^3\hat{\sigma}^2_{ki}\sigma^2_i\lambda_j(\hat{\sigma}^2_{ji}\lambda_0 + \sigma^2_i\lambda_j)\right]\frac{1}{(\hat{\sigma}^2_{ki}\lambda_0 + 2\sigma^2_i\lambda_k)} \tag{24}$$
$$B = \rho^2\lambda_0\hat{\sigma}^2_{ji} \tag{25}$$
$$C = -\rho\lambda_0 \tag{26}$$

A.3.2 Proof of Lemma 1

. With the restriction that $\lambda_k = 1 - \lambda_0 - \lambda_j$, it is sufficient to show that A is increasing in λ_j.

$$
\begin{aligned}
A &= \frac{(1-\lambda_0-\lambda_j)\lambda_j\rho^3\sigma_i^4\hat{\sigma}_{ji}^2+\rho^3\hat{\sigma}_{ki}^2\sigma_i^2\lambda_j(\hat{\sigma}_{ji}^2\lambda_0+\sigma_i^2\lambda_j)}{(\hat{\sigma}_{ki}^2\lambda_0+2\sigma_i^2(1-\lambda_0-\lambda_j))} \\
B &= \rho^2\lambda_0\hat{\sigma}_{ji}^2 \\
C &= -\rho\lambda_0 \\
A &\propto \frac{(1-\lambda_0-\lambda_j)\lambda_j\sigma_i^2\hat{\sigma}_{ji}^2+\hat{\sigma}_{ki}^2\lambda_j\left(\hat{\sigma}_{ji}^2\lambda_0+\sigma_i^2\lambda_j\right)}{\hat{\sigma}_{ki}^2\lambda_0+2\sigma_i^2\left(1-\lambda_0-\lambda_j\right)} \\
\frac{\partial A}{\partial\lambda_j} &= \left[\left((1-\lambda_0-\lambda_j)\lambda_j\sigma_i^2\hat{\sigma}_{ji}^2+\hat{\sigma}_{ki}^2\lambda_j\left(\hat{\sigma}_{ji}^2\lambda_0+\sigma_i^2\lambda_j\right)\right)'\times\left(\hat{\sigma}_{ki}^2\lambda_0+2\sigma_i^2\left(1-\lambda_0-\lambda_j\right)\right)\right. \\
&\quad \left.-\left((1-\lambda_0-\lambda_j)\lambda_j\sigma_i^2\hat{\sigma}_{ji}^2+\hat{\sigma}_{ki}^2\lambda_j\left(\hat{\sigma}_{ji}^2\lambda_0+\sigma_i^2\lambda_j\right)\right)\times\left(\hat{\sigma}_{ki}^2\lambda_0+2\sigma_i^2\left(1-\lambda_0-\lambda_j\right)\right)'\right] \\
&\quad \times\frac{1}{\left(\hat{\sigma}_{ki}^2\lambda_0+2\sigma_i^2\left(1-\lambda_0-\lambda_j\right)\right)^2} \\
&\propto \left(\sigma_i^2\hat{\sigma}_{ji}^2\hat{\sigma}_{ki}^2\lambda_0\left(1-\lambda_0-2\lambda_j\right)+2\sigma_i^4\hat{\sigma}_{ji}^2\left(1-\lambda_0-\lambda_j\right)\left(1-\lambda_0-2\lambda_j\right)+\hat{\sigma}_{ki}^4\hat{\sigma}_{ji}^2\lambda_0\lambda_0\right. \\
&\quad \left.+2\sigma_i^2\hat{\sigma}_{ki}^2\hat{\sigma}_{ji}^2\lambda_0\left(1-\lambda_0-\lambda_j\right)+2\sigma_i^2\hat{\sigma}_{ki}^2\lambda_j\left(\hat{\sigma}_{ki}^2\lambda_0+2\sigma_i^2\left(1-\lambda_0-\lambda_j\right)\right)\right) \\
&\quad +2\left(1-\lambda_0-\lambda_j\right)\lambda_j\sigma_i^4\hat{\sigma}_{ji}^2+2\sigma_i^2\hat{\sigma}_{ki}^2\hat{\sigma}_{ji}^2\lambda_0\lambda_j+2\sigma_i^4\hat{\sigma}_{ki}^2\lambda_j^2 \\
&= \sigma_i^2\hat{\sigma}_{ji}^2\hat{\sigma}_{ki}^2\lambda_0\left(3-3\lambda_0-2\lambda_j\right)+2\sigma_i^4\hat{\sigma}_{ji}^2\left(1-\lambda_0-\lambda_j\right)^2+\hat{\sigma}_{ki}^4\hat{\sigma}_{ji}^2\lambda_0\lambda_0 \\
&\quad +2\sigma_i^2\hat{\sigma}_{ki}^4\lambda_j\lambda_0+2\sigma_i^4\hat{\sigma}_{ki}^2\lambda_j\left(2-2\lambda_0-\lambda_j\right) \\
&> 0 \\
\text{Therefore} &\quad \frac{\partial\beta_{ji}}{\partial\lambda_j}<0.
\end{aligned}
$$

□

A.3.3 Proof of Proposition 4

Part 1:

Price informativeness in the general model is:

$$
PI_i=\frac{cov(p_i,z_i)}{\sigma_{pi}}=\frac{\sigma_i\sum_{j=1}^{l}\omega_{ji}\frac{\alpha_{ji}-1}{\alpha_{ji}}}{\sqrt{\frac{\sigma_{xi}^2}{\sigma_i^2}+\left[\sum_{j=1}^{l}\omega_{ji}\frac{\alpha_{ji}-1}{\alpha_{ji}}\right]^2+\sum_{j=1}^{l}\omega_{ij}^2\frac{\alpha_{ji}-1}{\alpha_{ji}^2}}}, \tag{27}
$$

where

$$
\omega_{ji}\equiv\frac{\partial\lambda_j q_{ji}}{\partial s_{ji}}=\lambda_j\beta_{1ji}
$$

Dividing the numerator and the denominator by $\sum\omega_{ji}$ gives:

$$
PI_i=\frac{\sigma_i\sum_{j=1}^{l}\frac{\omega_{ji}}{\sum\omega_{ji}}\frac{\alpha_{ji}-1}{\alpha_{ji}}}{\sqrt{\frac{\sigma_{xi}^2}{\sigma_i^2}\frac{1}{(\sum\omega_{ji})^2}+\left[\sum_{j=1}^{l}\frac{\omega_{ji}}{\sum\omega_{ji}}\frac{\alpha_{ji}-1}{\alpha_{ji}}\right]^2+\sum_{j=1}^{l}(\frac{\omega_{ji}}{(\sum\omega_{ji}})^2\frac{\alpha_{ji}-1}{\alpha_{ji}^2}}}, \tag{28}
$$

For part 1 of the proposition, we consider a symmetric exogenous information allocation, denoted as α_i, which simplifies PI to:

$$PI_i = \frac{\sigma_i \frac{\alpha_i - 1}{\alpha_i}}{\sqrt{\frac{\sigma_{xi}^2}{\sigma_i^2}\frac{1}{(\sum \omega_{ji})^2} + \left[\frac{\alpha_{ji}-1}{\alpha_{ji}}\right]^2 + \frac{\alpha_i - 1}{\alpha_i^2}\sum_{j=1}^{l}(\frac{\omega_{ji}}{(\sum \omega_{ji})})^2}}, \tag{29}$$

Clearly, the last term in the denominator is minimized for $\lambda_j = \lambda_k$. The question remains on what is the behavior of the function

$$\Omega = \sum_j \omega_{ji} = \lambda_j \beta_{ji} + (1 - \lambda_0 - \lambda_j)\beta_{ki}$$

Given the expressions (24)-(26), we have:

$$\Omega_i = \frac{-(2(-1+\lambda_j+\lambda_0)\sigma_i^2 - \lambda_0\sigma_j^2)(\lambda_0\rho^2\sigma_j^2 - \sqrt{\frac{(\lambda_0\rho^4\sigma_j^2(2\lambda_j\sigma_i^2+\lambda_0\sigma_j^2)(-2(-1+\lambda_0)\sigma_i^2+\lambda_0\sigma_j^2))}{(-2(-1+\lambda_j+\lambda_0)\sigma_i^2+\lambda_0\sigma_j^2)}})}{(2\rho^3\sigma_i^2\sigma_j^2((-1+\lambda_0)\sigma_i^2 - \lambda_0\sigma_j^2))} + \frac{(2\lambda_j\sigma_i^2+\lambda_0\sigma_j^2)(\lambda_0\rho^2\sigma_j^2 - \sqrt{\frac{(\lambda_0\rho^4\sigma_j^2(-2(-1+\lambda_0)\sigma_i^2+\lambda_0\sigma_j^2)(-2(-1+\lambda_j+\lambda_0)\sigma_i^2+\lambda_0\sigma_j^2))}{(2\lambda_j\sigma_i^2+\lambda_0\sigma_j^2)}})}{(2\rho^3\sigma_i^2\sigma_j^2((-1+\lambda_0)\sigma_i^2 - \lambda_0\sigma_j^2))}$$

The derivative of the expression with respect to λ_j is:

$$\begin{aligned}
\frac{d\Omega_i}{d\lambda_j} &= \frac{1}{(2\rho^3\sigma_i^2\sigma_{ji}^2((-1+\lambda_0)\sigma_i^2 - \lambda_0\sigma_{ji}^2))} \times \\
&\frac{-2\sigma_i^2(\sigma_i^2 - \lambda_0\sigma_i^2 + \lambda_0\sigma_{ji}^2)\sqrt{\frac{(\lambda_0\rho^4\sigma_{ji}^2(2\lambda_j\sigma_i^2+\lambda_0\sigma_{ji}^2)(-2(-1+\lambda_0)\sigma_i^2+\lambda_0\sigma_{ji}^2))}{(-2(-1+\lambda_j+\lambda_0)\sigma_i^2+\lambda_0\sigma_{ji}^2)}}}{2\lambda_j\sigma_i^2+\lambda_0\sigma_{ji}^2} \\
&+\frac{2\sigma_i^2(\sigma_i^2 - \lambda_0\sigma_i^2 + \lambda_0\sigma_{ji}^2)\sqrt{\frac{(\lambda_0\rho^4\sigma_{ji}^2(-2(-1+\lambda_0)\sigma_i^2+\lambda_0\sigma_{ji}^2)(-2(-1+\lambda_j+\lambda_0)\sigma_i^2+\lambda_0\sigma_{ji}^2))}{(2\lambda_j\sigma_i^2+\lambda_0\sigma_{ji}^2)}})}{(-2(-1+\lambda_j+\lambda_0)\sigma_i^2+\lambda_0\sigma_{ji}^2)} \\
&-2\sigma_i^2(\lambda_0\rho^2\sigma_{ji}^2 - \sqrt{\frac{(\lambda_0\rho^4\sigma_{ji}^2(2\lambda_j\sigma_i^2+\lambda_0\sigma_{ji}^2)(-2(-1+\lambda_0)\sigma_i^2+\lambda_0\sigma_{ji}^2))}{(-2(-1+\lambda_j+\lambda_0)\sigma_i^2+\lambda_0\sigma_{ji}^2)}}) \\
&+2\sigma_i^2(\lambda_0\rho^2\sigma_{ji}^2 - \sqrt{\frac{(\lambda_0\rho^4\sigma_{ji}^2(-2(-1+\lambda_0)\sigma_i^2+\lambda_0\sigma_{ji}^2)(-2(-1+\lambda_j+\lambda_0)\sigma_i^2+\lambda_0\sigma_{ji}^2))}{(2\lambda_j\sigma_i^2+\lambda_0\sigma_{ji}^2)}})
\end{aligned}$$

In the above expression, the denominator (first expression) is always negative, so the sign of the derivative is negatively related to (after dividing the above by $2\sigma_i^4$ and simplifying):

$$\begin{aligned}
\frac{d\Omega_i}{d\lambda_j} \propto &\frac{(-1+2\lambda_j+\lambda_0)\sigma_i^2\sqrt{\frac{(\lambda_0\rho^4\sigma_{ji}^2(2\lambda_j\sigma_i^2+\lambda_0\sigma_{ji}^2)(-2(-1+\lambda_0)\sigma_i^2+\lambda_0\sigma_{ji}^2))}{(-2(-1+\lambda_j+\lambda_0)\sigma_i^2+\lambda_0\sigma_{ji}^2)}}}{(2\lambda_j\sigma_i^2+\lambda_0\sigma_{ji}^2)} \\
&+\frac{((-1+2\lambda_j+\lambda_0)\sigma_i^2\sqrt{\frac{(\lambda_0\rho^4\sigma_{ji}^2(-2(-1+\lambda_0)\sigma_i^2+\lambda_0\sigma_{ji}^2)(-2(-1+\lambda_j+\lambda_0)\sigma_i^2+\lambda_0\sigma_{ji}^2))}{(2\lambda_j\sigma_i^2+\lambda_0\sigma_{ji}^2)}})}{(-2(-1+\lambda_j+\lambda_0)\sigma_i^2+\lambda_0\sigma_{ji}^2)}
\end{aligned}$$

Since both $(2\lambda_j\sigma_i^2+\lambda_0\sigma_{ji}^2)$ and $(-2(-1+\lambda_j+\lambda_0)\sigma_i^2+\lambda_0\sigma_{ji}^2)$ are strictly positive, the sign of the derivative depends negatively on the sign of $-1+2\lambda_j+\lambda_0$, which is negative for $\lambda_j < \frac{1-\lambda_0}{2}$, positive for $\lambda_j > \frac{1-\lambda_0}{2}$ and zero for $\lambda_j = \frac{1-\lambda_0}{2}$.
Hence, Ω_i is increasing for $\lambda_j < \frac{1-\lambda_0}{2}$, and decreasing for $\lambda_j > \frac{1-\lambda_0}{2}$, with a global maximum at $\lambda_j = \frac{1-\lambda_0}{2}$, which is equivalent to $\lambda_j = \lambda_k$.

Part 2:

The definition of PI when $\alpha_j \neq \alpha_i$ is:

$$PI = \frac{\sigma_i\left(\lambda_j \beta_{1ji}\frac{\alpha_{ji}-1}{\alpha_{ji}} + (1-\lambda_0-\lambda_j)\beta_{1ki}\frac{\alpha_{ki}-1}{\alpha_{ki}}\right)}{\sqrt{\frac{\sigma_{xi}^2}{\sigma_i^2} + \left[\lambda_j \beta_{1ji}\frac{\alpha_{ji}-1}{\alpha_{ji}} + (1-\lambda_0-\lambda_j)\beta_{1ki}\frac{\alpha_{ki}-1}{\alpha_{ki}}\right]^2 + \left(\lambda_j^2 \beta_{1ji}^2\frac{\alpha_{ji}-1}{\alpha_{ji}^2} + (1-\lambda_0-\lambda_j)^2\beta_{1ki}^2\frac{\alpha_{ki}-1}{\alpha_{ki}^2}\right)}}$$

The first order condition is with respect to λ_j:

$$\begin{aligned}
\sigma_i X &= \frac{\left(\sigma_i\left(\lambda_j \beta_{1ji}\frac{\alpha_{ji}-1}{\alpha_{ji}} + (1-\lambda_0-\lambda_j)\beta_{1ki}\frac{\alpha_{ki}-1}{\alpha_{ki}}\right)\right)^3}{PI^2}\left(\left[\lambda_j \beta_{1ji}\frac{\alpha_{ji}-1}{\alpha_{ji}} + (1-\lambda_0-\lambda_j)\beta_{1ki}\frac{\alpha_{ki}-1}{\alpha_{ki}}\right] X\right. \\
&\left. + \left(\left(\lambda_j \beta_{1ji}^2 + \lambda_j^2 \beta_{1ji}\beta_{1ji}^{'}\right)\frac{\alpha_{ji}-1}{\alpha_{ji}^2} + \left(-(1-\lambda_0-\lambda_j)\beta_{1ki}^2 + (1-\lambda_0-\lambda_j)^2\beta_{1ki}\beta_{1ki}^{'}\right)\frac{\alpha_{ki}-1}{\alpha_{ki}^2}\right)\right) \\
X &\equiv \left(\left(\beta_{1ji} + \lambda_j \beta_{1ji}^{'}\right)\frac{\alpha_{ji}-1}{\alpha_{ji}} + \left(-\beta_{1ki} + (1-\lambda_0-\lambda_j)\beta_{1ki}^{'}\right)\frac{\alpha_{ki}-1}{\alpha_{ki}}\right)
\end{aligned}$$

Where $\beta_{1ki}' = \frac{\partial \beta_{1ki}}{\partial \lambda_j}$ and $\beta_{1ji}' = \frac{\partial \beta_{1ji}}{\partial \lambda_j}$. If we set $\lambda_j = \lambda_k = 1 - \lambda_j - \lambda_0$, the expression can simplify to:

$$\begin{aligned}
0 &= \left(\beta_{1ji}\frac{\alpha_{ji}-1}{\alpha_{ji}} + \beta_{1ki}\frac{\alpha_{ki}-1}{\alpha_{ki}}\right)^3 \left(\left[\beta_{1ji}\frac{\alpha_{ji}-1}{\alpha_{ji}} + \beta_{1ki}\frac{\alpha_{ki}-1}{\alpha_{ki}}\right] X\right. \\
&\left. + \left(\beta_{1ji}\left(\beta_{1ji} + \lambda_j \beta_{1ji}^{'}\right)\frac{\alpha_{ji}-1}{\alpha_{ji}^2} + \beta_{1ki}\left(-\beta_{1ki} + \lambda_j \beta_{1ki}^{'}\right)\frac{\alpha_{ki}-1}{\alpha_{ki}^2}\right)\right) \\
&- X\frac{\left(\beta_{1ji}\frac{\alpha_{ji}-1}{\alpha_{ji}} + \beta_{1ki}\frac{\alpha_{ki}-1}{\alpha_{ki}}\right)^2}{\frac{\sigma_{xi}^2}{\sigma_i^2} + \lambda_j^2\left[\beta_{1ji}\frac{\alpha_{ji}-1}{\alpha_{ji}} + \beta_{1ki}\frac{\alpha_{ki}-1}{\alpha_{ki}}\right]^2 + \lambda_j^2\left(\beta_{1ji}^2\frac{\alpha_{ji}-1}{\alpha_{ji}^2} + \beta_{1ki}^2\frac{\alpha_{ki}-1}{\alpha_{ki}^2}\right)} \\
0 &= \left(\beta_{1ji}\frac{\alpha_{ji}-1}{\alpha_{ji}} + \beta_{1ki}\frac{\alpha_{ki}-1}{\alpha_{ki}}\right)\left(\left[\beta_{1ji}\frac{\alpha_{ji}-1}{\alpha_{ji}} + \beta_{1ki}\frac{\alpha_{ki}-1}{\alpha_{ki}}\right]\right. \\
&\left(\left(\beta_{1ji} + \lambda_j \beta_{1ji}^{'}\right)\frac{\alpha_{ji}-1}{\alpha_{ji}} + \left(-\beta_{1ki} + \lambda_j \beta_{1ki}^{'}\right)\frac{\alpha_{ki}-1}{\alpha_{ki}}\right) \\
&\left. + \left(\beta_{1ji}\left(\beta_{1ji} + \lambda_j \beta_{1ji}^{'}\right)\frac{\alpha_{ji}-1}{\alpha_{ji}^2} + \beta_{1ki}\left(-\beta_{1ki} + \lambda_j \beta_{1ki}^{'}\right)\frac{\alpha_{ki}-1}{\alpha_{ki}^2}\right)\right) \\
&- \frac{\left(\left(\beta_{1ji} + \lambda_j \beta_{1ji}^{'}\right)\frac{\alpha_{ji}-1}{\alpha_{ji}} + \left(-\beta_{1ki} + \lambda_j \beta_{1ki}^{'}\right)\frac{\alpha_{ki}-1}{\alpha_{ki}}\right)}{\frac{\sigma_{xi}^2}{\sigma_i^2} + \lambda_j^2\left[\beta_{1ji}\frac{\alpha_{ji}-1}{\alpha_{ji}} + \beta_{1ki}\frac{\alpha_{ki}-1}{\alpha_{ki}}\right]^2 + \lambda_j^2\left(\beta_{1ji}^2\frac{\alpha_{ji}-1}{\alpha_{ji}^2} + \beta_{1ki}^2\frac{\alpha_{ki}-1}{\alpha_{ki}^2}\right)} \\
&= F(\alpha_{ji}, \alpha_{ki}, \lambda_0, \lambda_j, \cdot) - G(\alpha_{ji}, \alpha_{ki}, \lambda_0, \lambda_j, \cdot, \sigma_{xi})
\end{aligned}$$

The function G in this expression is a function of σ_{xi}^2, while the function F is not. Therefore, while there could be a set of parameter values that satisfy the above expression, in order for there to be a general solution, it must be that the case that both F and G are equal to zero, as they will not be equal to each other outside of a measure-0 set of values for σ^2_{xi}. Therefore, the conditions to be satisfied become:

$$\begin{aligned}
0 &= \left(\beta_{1ji}+\lambda_j\beta'_{1ji}\right)\frac{\alpha_{ji}-1}{\alpha_{ji}}+\left(-\beta_{1ki}+\lambda_j\beta'_{1ki}\right)\frac{\alpha_{ki}-1}{\alpha_{ki}} \\
0 &= \left(\beta_{1ji}\frac{\alpha_{ji}-1}{\alpha_{ji}}+\beta_{1ki}\frac{\alpha_{ki}-1}{\alpha_{ki}}\right)\left(\left[\beta_{1ji}\frac{\alpha_{ji}-1}{\alpha_{ji}}+\beta_{1ki}\frac{\alpha_{ki}-1}{\alpha_{ki}}\right]\right. \\
&\quad \left(\left(\beta_{1ji}+\lambda_j\beta'_{1ji}\right)\frac{\alpha_{ji}-1}{\alpha_{ji}}+\left(-\beta_{1ki}+\lambda_j\beta'_{1ki}\right)\frac{\alpha_{ki}-1}{\alpha_{ki}}\right) \\
&\quad \left.+\left(\beta_{1ji}\left(\beta_{1ji}+\lambda_j\beta'_{1ji}\right)\frac{\alpha_{ji}-1}{\alpha^2_{ji}}+\beta_{1ki}\left(-\beta_{1ki}+\lambda_j\beta'_{1ki}\right)\frac{\alpha_{ki}-1}{\alpha^2_{ki}}\right)\right)
\end{aligned}$$

$\left(\beta_{1ji}\frac{\alpha_{ji}-1}{\alpha_{ji}}+\beta_{1ki}\frac{\alpha_{ki}-1}{\alpha_{ki}}\right)$ is strictly positive, so the two conditions are :

$$\begin{aligned}
0 &= \left(\beta_{1ji}+\lambda_j\beta'_{1ji}\right)\frac{\alpha_{ji}-1}{\alpha_{ji}}+\left(-\beta_{1ki}+\lambda_j\beta'_{1ki}\right)\frac{\alpha_{ki}-1}{\alpha_{ki}} \\
0 &= \left(\beta_{1ji}\left(\beta_{1ji}+\lambda_j\beta'_{1ji}\right)\frac{\alpha_{ji}-1}{\alpha^2_{ji}}+\beta_{1ki}\left(-\beta_{1ki}+\lambda_j\beta'_{1ki}\right)\frac{\alpha_{ki}-1}{\alpha^2_{ki}}\right)\Bigg)
\end{aligned}$$

Suppose that we assume that the first condition is always satisfied. Then we can rewrite the second condition as:

$$\frac{\beta_{ji}}{\alpha_{ji}} = \frac{\beta_{ki}}{\alpha_{ki}}$$

Rearranging gives the necessary condition for symmetric λs to maximize PI for asymmetric learning:

$$\frac{\beta_{ji}}{\beta_{ki}} = \frac{\alpha_{ji}}{\alpha_{ki}}$$

Consider the left hand size of the above expression. Plugging in the solution using expressions (24)-(26) for duopolist j and an analogous set of equations for duopolist k gives:

$$\frac{\beta_{ji}}{\beta_{ki}} = \frac{((1-\lambda_0)\sigma^2_i+\lambda_0\hat{\sigma}^2_{ji})(\lambda_0\rho^2\hat{\sigma}^2_{ki}-\sqrt{\frac{\lambda_0\rho^4((1-\lambda_0)\sigma^2_i+\lambda_0\hat{\sigma}^2_{ki})(\lambda_0\hat{\sigma}^2_{ji}\hat{\sigma}^2_{ki}+(1-\lambda_0)\sigma^2_i(\hat{\sigma}^2_{ji}+\hat{\sigma}^2_{ki}))}{(1-\lambda_0)\sigma^2_i+\lambda_0\hat{\sigma}^2_{ji}}})}{((1-\lambda_0)\sigma^2_i+\lambda_0\hat{\sigma}^2_{ki})(\lambda_0\rho^2\hat{\sigma}^2_{ji}-\sqrt{\frac{\lambda_0\rho^4((1-\lambda_0)\sigma^2_i+\lambda_0\hat{\sigma}^2_{ji})(\lambda_0\hat{\sigma}^2_{ji}\hat{\sigma}^2_{ki}+(1-\lambda_0)\sigma^2_i(\hat{\sigma}^2_{ji}+\hat{\sigma}^2_{ki}))}{(1-\lambda_0)\sigma^2_i+\lambda_0\hat{\sigma}^2_{ki}}})} \tag{30}$$

Consider first the numerator of this expression. Multiplying through by the first expression:

$$((1-\lambda_0)\sigma^2_i+\lambda_0\hat{\sigma}^2_{ji})\lambda_0\rho^2\hat{\sigma}^2_{ki}-\sqrt{\lambda_0\rho^4((1-\lambda_0)\sigma^2_i+\lambda_0\hat{\sigma}^2_{ji})((1-\lambda_0)\sigma^2_i+\lambda_0\hat{\sigma}^2_{ki})(\lambda_0\hat{\sigma}^2_{ji}\hat{\sigma}^2_{ki}+(1-\lambda_0)\sigma^2_i(\hat{\sigma}^2_{ji}+\hat{\sigma}^2_{ki}))}$$

Factoring out ρ^2 and λ_0 and rearranging gives:

$$\lambda_0(1-\lambda_0)\alpha_{ji}\hat{\sigma}^2_{ji}\hat{\sigma}^2_{ki}+\lambda^2_0\hat{\sigma}^2_{ji}\hat{\sigma}^2_{ki}-\lambda^2_0\sqrt{(\frac{1-\lambda_0}{\lambda_0}\sigma^2_i+\hat{\sigma}^2_{ji})(\frac{1-\lambda_0}{\lambda_0}\sigma^2_i+\hat{\sigma}^2_{ki})(\hat{\sigma}^2_{ji}\hat{\sigma}^2_{ki}+\frac{1-\lambda_0}{\lambda_0}\sigma^2_i(\hat{\sigma}^2_{ji}+\hat{\sigma}^2_{ki}))} \tag{31}$$

Performing analogous calculations for the denominator of (30), we get

$$\lambda_0(1-\lambda_0)\alpha_{ki}\hat{\sigma}^2_{ji}\hat{\sigma}^2_{ki}+\lambda^2_0\hat{\sigma}^2_{ji}\hat{\sigma}^2_{ki}-\lambda^2_0\sqrt{(\frac{1-\lambda_0}{\lambda_0}\sigma^2_i+\hat{\sigma}^2_{ji})(\frac{1-\lambda_0}{\lambda_0}\sigma^2_i+\hat{\sigma}^2_{ki})(\hat{\sigma}^2_{ji}\hat{\sigma}^2_{ki}+\frac{1-\lambda_0}{\lambda_0}\sigma^2_i(\hat{\sigma}^2_{ji}+\hat{\sigma}^2_{ki}))} \tag{32}$$

For the ratio of (31) and (32) to be equal to $\frac{\alpha_{ji}}{\alpha_{ki}}$ in the case of $\alpha_{ji} \neq \alpha_{ki}$, it has to be that

$$\lambda_0^2\hat{\sigma}_{ji}^2\hat{\sigma}_{ki}^2 - \lambda_0^2\sqrt{(\frac{1-\lambda_0}{\lambda_0}\sigma_i^2 + \hat{\sigma}_{ji}^2)(\frac{1-\lambda_0}{\lambda_0}\sigma_i^2 + \hat{\sigma}_{ki}^2)(\hat{\sigma}_{ji}^2\hat{\sigma}_{ki}^2 + \frac{1-\lambda_0}{\lambda_0}\sigma_i^2(\hat{\sigma}_{ji}^2 + \hat{\sigma}_{ki}^2))} = 0$$

Factoring out $\lambda_0^2\hat{\sigma}_{ji}^2\hat{\sigma}_{ki}^2$, this condition boils down to

$$(\frac{1-\lambda_0}{\lambda_0}\alpha_{ji} + 1)(\frac{1-\lambda_0}{\lambda_0}\alpha_{ki} + 1)(1 + \frac{1-\lambda_0}{\lambda_0}\sigma_i^2\frac{\hat{\sigma}_{ji}^2 + \hat{\sigma}_{ki}^2}{\hat{\sigma}_{ji}^2\hat{\sigma}_{ki}^2}) = 1$$

which is never satisfied.

Part 3:

For Part 3, observe that for the case of $\alpha_{ji} = 1$ and $\alpha_{ki} > 1$, we have:

$$PI_i = \frac{\sigma_i\frac{\alpha_{ki}-1}{\alpha_{ki}}}{\sqrt{\frac{\sigma_{xi}^2}{\sigma_i^2}\frac{1}{\omega_{ki}^2} + \left[\frac{\alpha_{ji}-1}{\alpha_{ji}}\right]^2 + \frac{\alpha_{ki}-1}{\alpha_{ki}^2}}}, \tag{33}$$

The question becomes which value of λ_j maximizes information pass-through ω_{ki}, where

$$\omega_{ki} = \lambda_k\beta_{ki} = (1-\lambda_0-\lambda_j)\beta_{ki},$$

and β_{ki} is determined by (24)-(26). In particular, the sign of the derivative of ω_{ki} with respect to λ_j depends on the sign of:

$$\begin{aligned}&\hat{\sigma}_{ki}^2((-2+\lambda_0)\sigma_i^2 - \lambda_0\hat{\sigma}_{ki}^2)\\&+[-\rho^2 4(-1+\lambda_j+\lambda_0)^3\sigma_i^6 + 2(-1+\lambda_j+\lambda_0)(4\lambda_j^2 + \lambda_0(-3+4\lambda_0) + \lambda_j(-2+8\lambda_0))\sigma_i^4\hat{\sigma}_{ki}^2\\&-(4\lambda_j^3 + 6\lambda_j(-1+2\lambda_j)\lambda_0 + 2(-2+7\lambda_j)\lambda_0^2 + 5\lambda_0^3)\sigma_i^2\hat{\sigma}_{ki}^4 + \lambda_0^2(2\lambda_j+\lambda_0)\hat{\sigma}_{ki}^6] \times\\&\frac{1}{(2\lambda_j+\lambda_0)\sqrt{\frac{(\lambda_0\rho^4(4(-1+\lambda_j+\lambda_0)^2\sigma_i^4 - 4(-1+\lambda_j+\lambda_0)(\lambda_j+\lambda_0)\sigma_i^2\hat{\sigma}_{ki}^2 + \lambda_0(2\lambda_j+\lambda_0)\hat{\sigma}_{ki}^4))}{(2\lambda_j+\lambda_0)}}}\end{aligned}$$

Consider first the limit case of $\lambda_j = 0$, the above function converges to

$$-4(-1+\lambda_0)^3\sigma_i^6 + 2(-1+\lambda_0)\lambda_0(-3+4\lambda_0)\sigma_i^4\sigma_{ki}^2 + (4-5\lambda_0)\lambda_0^2\sigma_i^2\sigma_{ki}^4 + \lambda_0^3\sigma_{ki}^6 \to_{\lambda_0\to 0} = 4\sigma_i^6 > 0$$

so that for small enough λ_0, it is always beneficial from the PI perspective to increase the size of the passive oligopolist. The reason for that is that the information pass-through of the active oligopolist is decreasing enough with their size that actually reducing their size increases PI. The conclusion is that if the fringe sector is small enough, an interior solution for λ_j maximizes PI.

A.3.4 Proof of Proposition 5

Proof. It is sufficient to show that $\frac{d\beta_{ji}}{d\hat{\sigma}_{ki}^2} < 0$, which is true if and only if $\frac{\partial A}{\partial\hat{\sigma}_{ki}^2} > 0$. Using (24):

$$\frac{\partial A}{\partial\hat{\sigma}_{ki}^2} = -\frac{(\lambda_j(-1+\lambda_j+\lambda_0)\sigma_i^4(2\lambda_j\sigma_i^2 + \lambda_0\sigma_{ji}^2)\rho^3)}{(-2(-1+\lambda_j+\lambda_0)\sigma_i^2 + \lambda_0\sigma_k^2)^2}$$

which is equal in sign to

$$-(-1+\lambda_j+\lambda_0) > 0.$$

□

A.4 Derivation of Equations (7)-(9)

The price observed by oligopolist k is

$$p_i \sum_{j=0}^{l} \lambda_j r \beta_{2ji} = -x_i + \lambda_k \beta_{1ki} s_{ki} + \sum_{j=0}^{l} \lambda_j \beta_{0ji} + \sum_{j=-k} \lambda_j \beta_{1ji} s_{ji}.$$

With $\Delta_i \equiv r \sum_{j=0}^{l} \lambda_j \beta_{2ji}$, we can write:

$$p_i = -\frac{1}{\Delta_i} x_i + \frac{1}{\Delta_i} \lambda_k \beta_{1ki} s_{ki} + \frac{1}{\Delta_i} \sum_{j=0}^{l} \lambda_j \beta_{0ji} + \frac{1}{\Delta_i} \sum_{j=-k} \lambda_j \beta_{1ji} s_{ji}$$

The conditional distribution of the signal s_{ji} is Normal, with mean $\bar{z} + (1 - \frac{1}{\alpha_{ji}})\varepsilon_i$ and variance $(1 - \frac{1}{\alpha_{ji}})\frac{1}{\alpha_{ji}}\sigma_i^2$, and hence, denoting $\zeta_{ji} \equiv s_{ji} - E(s_{ji}|z_i)$, we can write:

$$cov_k(z_i, p_i) = \frac{1}{\Delta_i} \sum_{j=-k} \lambda_j \beta_{1ji} cov_k(s_{ji}, z_i) = \frac{1}{\Delta_i} \sum_{j=-k} \lambda_j \beta_{1ji} cov_k((1 - \frac{1}{\alpha_{ji}})\varepsilon_i + \zeta_{ji}, \varepsilon_i) =$$

$$= \frac{1}{\Delta_i} \sum_{j=-k} \lambda_j \beta_{1ji} cov_k((1 - \frac{1}{\alpha_{ji}})\varepsilon_i, \varepsilon_i) = \frac{1}{\Delta_i} \sum_{j=-k} \lambda_j \beta_{1ji} (1 - 1/\alpha_{ji}) \eta_{ki}^2$$

$$= \frac{1}{\Delta_i} \frac{1}{\alpha_{ki}} \sum_{j=-k} \lambda_j \beta_{1ji} (1 - 1/\alpha_{ji}) \sigma_i^2$$

We note that the covariance of oligopolist k depends both on his own learning choices as well as the learning choices of the other oligopolists. This is also true for the variance of the price:

$$var_k(p_i) = (\frac{1}{\Delta_i})^2 \sigma_{xi}^2 + (\frac{1}{\Delta_i})^2 var_k(\sum_{j=-k} \lambda_j \beta_{1ji} (\bar{z}_i + (1 - \frac{1}{\alpha_{ji}})\varepsilon_i + \zeta_{ji})) =$$

$$= (\frac{1}{\Delta_i})^2 \sigma_{xi}^2 + (\frac{1}{\Delta_i})^2 \frac{1}{\alpha_{ki}} \sigma_i^2 \left[\sum_{j=-k} \lambda_j \beta_{1ji} (1 - \frac{1}{\alpha_{ji}}) \right]^2 + (\frac{1}{\Delta_i})^2 \sum_{j=-k} \lambda_j^2 \beta_{1ji}^2 (1 - \frac{1}{\alpha_{ji}}) \frac{1}{\alpha_{ji}} \sigma_i^2)$$

$E_{ki}[p_i|s_{ki}]$ is given by (omitting the conditioning on the signal notation)

$$E_{ki}[p_i] = -\frac{1}{\Delta_i} \bar{x}_i + \frac{1}{\Delta_i} \lambda_k \beta_{1ki} s_{ki} + \frac{1}{\Delta_i} \sum_{j=0}^{l} \lambda_j \beta_{0ji} + \frac{1}{\Delta_i} \sum_{j=-k} \lambda_j \beta_{1ji} (\bar{z}_i + (1 - 1/\alpha_{ji})(s_{ki} - \bar{z}_i))$$

$$E_{ki}[p_i] = s_{ki} \frac{1}{\Delta_i} \left[\lambda_k \beta_{1ki} + \sum_{j \neq k} \lambda_j \beta_{1ji} (1 - 1/\alpha_{ji}) \right] - \frac{1}{\Delta_i} \bar{x}_i + \frac{1}{\Delta_i} \sum_{j=1}^{l} \lambda_j \beta_{0ji} + \frac{1}{\Delta_i} \sum_{j=-k} \lambda_j \beta_{1ji} 1/\alpha_{ji} \bar{z}_i$$

Denote:

$$\Gamma_{ki} = -\frac{1}{\Delta_i} \bar{x}_i + \frac{1}{\Delta_i} \sum_{j=0}^{l} \lambda_j \beta_{0ji} + \frac{1}{\Delta_i} \sum_{j=-k} \lambda_j \beta_{1ji} 1/\alpha_{ji} \bar{z}_i$$

and

$$\theta_{ki} = \frac{1}{\Delta_i}\left[\lambda_k\beta_{1ki} + \sum_{j\neq k}\lambda_j\beta_{1ji}(1-1/\alpha_{ji})\right]$$

Plugging these results in (3), we get

$$q_{ji} = \frac{\mu_{ji} - rp_i}{\rho\hat{\sigma}^2_{ji} + r\frac{dp_i}{dq_{ji}}},$$

$$\mu_{ji} = s_{ji} + \frac{cov_j(z_i,p_i)}{var_j(p_i)}(p_i - E_{ji}[p_i])$$

$$\hat{\sigma}^2_{ji} = \frac{1}{\alpha_{ji}}\sigma_i^2 - \frac{cov_j^2(z_i,p_i)}{var_j(p_i)}$$

With $\gamma_{ji} \equiv cov_j(p_i)/\sigma^2_{pji}$, we can further write

$$q_{ji}(\rho\hat{\sigma}^2_{ji} + r\frac{dp_i}{dq_{ji}}) = s_{ji} + \gamma_{ji}p_i - rp_i - \gamma_{ji}(s_{ji}\theta_{ji} + \Gamma_{ji})$$

$$q_{ji}(\rho\hat{\sigma}^2_{ji} + r\frac{dp_i}{dq_{ji}}) = -\gamma_{ji}\Gamma_{ji} + s_{ji}(1-\gamma_{ji}\theta_{ji}) - r(1-\gamma_{ji}/r)p_i$$

Given that and equation (1), we obtain the fixed point for betas:

$$\begin{aligned}
\beta_{0ji} &= \frac{-\gamma_{ji}\Gamma_{ji}}{\rho\hat{\sigma}^2_{ji} + r\frac{dp_i}{dq_{ji}}} \\
\beta_{1ji} &= \frac{(1-\gamma_{ji}\theta_{ji})}{\rho\hat{\sigma}^2_{ji} + r\frac{dp_i}{dq_{ji}}}, \\
\beta_{2ji} &= \frac{1-\gamma_{ji}/r}{\rho\hat{\sigma}^2_{ji} + r\frac{dp_i}{dq_{ji}}}, \\
\frac{dp_i}{dq_{ki}} &= \frac{\lambda_k}{\sum_{j=-k}\lambda_j r\beta_{2ji}}.
\end{aligned}$$

A.5 Utility Maximization

The ex-ante information decision follows the maximization problem:

$$E_0U_j = \sum_{i=1}^{n} E_0(\hat{\mu}_{ji} - rp_i)^2 \frac{\frac{\rho}{2}\hat{\sigma}^2_{ji} + r\frac{dp_i}{dq_{ji}}}{(\rho\hat{\sigma}^2_{ji} + r\frac{dp_i}{dq_{ji}})^2} \tag{34}$$

Using market clearing:

$$p_i = -\frac{1}{\Delta_i}x_i + \frac{1}{\Delta_i}\sum_{j=0}^{l}\lambda_j\beta_{0ji} + \frac{1}{\Delta_i}\sum_{j=0}^{l}\lambda_j\beta_{1ji}s_{ji},$$

where $\Delta_i \equiv r\sum_{j=0}^{l}\lambda_j\beta_{2ji}$, we compute $E_{ji}(\mu_{ji} - rp_i)^2 = \widehat{R}_i^2 + \widehat{V}_{ji}$, where $\widehat{R}_i$ and $\widehat{V}_{ji}$ denote the ex-ante mean and variance of expected excess returns,

$$\hat{R}_i = E_{ji}(\mu_{ji} - rp_i) = \frac{r}{\Delta_i}x_i - \frac{r}{\Delta_i}\sum_{j=0}^{l}\lambda_j\beta_{0ji} + \bar{z} - \frac{r}{\Delta_i}\sum_{j=0}^{l}\lambda_j\beta_{1ji}\bar{z}$$

$$= \frac{r}{\Delta_i} x_i - \frac{r}{\Delta_i} \sum_{j=0}^{l} \lambda_j \beta_{0ji} + \bar{z}(1 - \frac{r}{\Delta_i} \sum_{j=0}^{l} \lambda_j \beta_{1ji})$$

We now compute

$$\hat{V}_{ji} = var(\mu - rp_i) = var(\mu_{ji}) + var(rp_i) - 2rcov(\mu_{ji}, p_i)$$

We obtain

$$\begin{aligned}
var(\mu_{ji}) &= var(s_{ji} + \gamma_{ji}(p_i - E_j[p_i])) \\
&= var(s_{ji}) + \gamma_{ji}^2 var(p_i) + \gamma_{ji}^2 var_j(E(p_i)) + 2\gamma_{ji} cov(s_{ji}, p_i) - 2\gamma_{ji} cov(s_{ji}, E_j(p_i)) - 2\gamma_{ji}^2 cov(p_i, E_j(p_i)) \\
var(rp_i) &= r^2 var(p_i) = \left(\frac{r}{\Delta_i}\right)^2 \left(\sigma_{ix}^2 + \sum_{k=1}^{l} \lambda_k^2 \beta_{1ki}^2 (1 - 1/\alpha_{ki}) \sigma_i^2\right) \\
2rcov(\mu_{ji}, p_i) &= 2rcov(s_{ji} + \gamma_{ji}(p_i - E_j[p_i]), p_i) = 2rcov(s_{ji}, p_i) + 2r\gamma_{ji} var(p_i) - 2r\gamma_{ji} cov(p_i, E_j(p_i))
\end{aligned}$$

Summing up:

$$\begin{aligned}
\hat{V}_{ji} &= var(s_{ji}) + \gamma_{ji}^2 var(p_i) + \gamma_{ji}^2 var_j(E(p_i)) + 2\gamma_{ji} cov(s_{ji}, p_i) - 2\gamma_{ji} cov(s_{ji}, E_j(p_i)) - 2\gamma_{ji}^2 cov(p_i, E_j(p_i)) + r^2 var(p_i) \\
&\quad -2r[cov(s_{ji}, p_i) + \gamma_{ji} var(p_i) - \gamma_{ji} cov(p_i, E_j(p_i))] \\
&= var(s_{ji}) + [\gamma_{ji}^2 + r^2 - 2r\gamma_{ji}] var(p_i) + \gamma_{ji}^2 var_j(E(p_i)) + 2[\gamma_{ji} - r] cov(s_{ji}, p_i) \\
&\quad -2\gamma_{ji} cov(s_{ji}, E_j(p_i)) - 2\gamma_{ji}[\gamma_{ji} - r] cov(p_i, E_j(p_i))
\end{aligned}$$

where

$$var(s_{ji}) = (1 - 1/\alpha_{ji})\sigma_i^2$$

$$var_j(p_i) = \left(\frac{1}{\Delta_i}\right)^2 \left(\sigma_{ix}^2 + \sum_{k=1}^{l} \lambda_k^2 \beta_{1ki}^2 (1 - 1/\alpha_{ki}) \sigma_i^2\right)$$

$$var_j(E(p_i)) = \theta_{ji}^2 (1 - 1/\alpha_{ji}) \sigma_i^2 = \theta_{ji}^2 var(s_{ji})$$

$$cov_j(s_{ji}, p_i) = \frac{1}{\Delta_i} \lambda_j \beta_{1ji} (1 - 1/\alpha_{ji}) \sigma_i^2 = \frac{1}{\Delta_i} \lambda_j \beta_{1ji} var(s_{ji})$$

$$cov_j(s_{ji}, E_j(p_i)) = \theta_{ji} (1 - 1/\alpha_{ji}) \sigma_i^2 = \theta_{ji} var(s_{ji})$$

$$cov_j(p_i, E_j(p_i)) = \theta_{ji} cov_j(s_{ji}, p_i) = \theta_{ji} \frac{1}{\Delta_i} \lambda_j \beta_{1ji} var(s_{ji})$$

Plugging in:

$$\hat{V}_{ji} = var(s_{ji})[1 + \theta_{ji}^2 \gamma_{ji}^2 + 2(\gamma_{ji} - r)\frac{1}{\Delta_i} \lambda_j \beta_{1ji} - 2\gamma_{ji}\theta_{ji} - 2\gamma_{ji}[\gamma_{ji} - r]\theta_{ji} \frac{1}{\Delta_i} \lambda_j \beta_{1ji}] + [\gamma_{ji} - r]^2 var(p_i)$$

Finally, since the competitive fringe investors have zero capacity, they only optimize along the quantity dimension, not the learning dimension, and hence it is not necessary to derive their ex-ante utility.

A.6 Derivation of Equation (10)

Using market clearing:

$$p_i = -\frac{1}{\Delta_i} x_i + \frac{1}{\Delta_i} \sum_{j=0}^{l} \lambda_j \beta_{0ji} + \frac{1}{\Delta_i} \sum_{j=0}^{l} \lambda_j \beta_{1ji} s_{ji} = -\frac{1}{\Delta_i} x_i + \frac{1}{\Delta_i} \sum_{j=0}^{l} \lambda_j \beta_{0ji} + \frac{1}{\Delta_i} \sum_{j=0}^{l} \lambda_j \beta_{1ji} [\bar{z}_i + (1 - \frac{1}{\alpha_{ji}}) \varepsilon_i + \zeta_{ji}], \tag{35}$$

Given that, we have

$$cov(p_i, z_i) = \frac{1}{\Delta_i} \sum_{j=0}^{l} \lambda_j \beta_{1ji} (1 - \frac{1}{\alpha_{ji}}) \sigma_i^2$$

and

$$var(p_i) = \sigma_i^2 \frac{1}{\Delta_i^2} \left[\frac{\sigma_{xi}^2}{\sigma_i^2} + \left(\sum_{j=0}^{l} \lambda_j \beta_{1ji} (1 - \frac{1}{\alpha_{ji}}) \right)^2 + \sum_{j=0}^{l} \lambda_j^2 \beta_{1ji}^2 \frac{\alpha_{ji} - 1}{\alpha_{ji}^2} \right]$$

Then, PI equals (the coefficient $\frac{1}{\Delta_i}$ cancels out and $\alpha_{0i} = 1$)

$$PI_i = \frac{cov(p_i, z_i)}{\sigma_{pi}} = \frac{\sigma_i \sum_{j=1}^{l} \lambda_j \beta_{1ji} \frac{\alpha_{ji} - 1}{\alpha_{ji}}}{\sqrt{\frac{\sigma_{xi}^2}{\sigma_i^2} + \left[\sum_{j=1}^{l} \lambda_j \beta_{1ji} (1 - \frac{1}{\alpha_{ji}}) \right]^2 + \sum_{j=1}^{l} \lambda_j^2 \beta_{1ji}^2 \frac{\alpha_{ji} - 1}{\alpha_{ji}^2}}}$$

Notice that $\lambda_j \beta_{1ji} = \frac{\partial \lambda_j q_{ji}}{\partial s_{ji}}$ is the reaction of the total quantity an oligopolist is purchasing with respect to the private signal, which we term the information pass-through. Defining information pass-through as

$$\omega_{ji} \equiv \frac{\partial \lambda_j q_{ji}}{\partial s_{ji}}.$$

results in equation (10)

$$PI_i = \frac{cov(p_i, z_i)}{\sigma_{pi}} = \frac{\sigma_i \sum_{j=1}^{l} \omega_{ji} \frac{\alpha_{ji} - 1}{\alpha_{ji}}}{\sqrt{\frac{\sigma_{xi}^2}{\sigma_i^2} + \left[\sum_{j=1}^{l} \omega_{ji} (1 - \frac{1}{\alpha_{ji}}) \right]^2 + \sum_{j=1}^{l} \omega_{ji}^2 \frac{\alpha_{ji} - 1}{\alpha_{ji}^2}}}.$$

A.7 Derivation of Equation (11)

Proof. It is enough to show that information pass-through:

$$\lambda_1 \beta_i = \frac{\lambda_1 (1 - \lambda_1) \beta_{20i} \alpha_i}{\rho (1 - \lambda_1) \sigma_i^2 \beta_{20i} + \lambda_1 \alpha_i}.$$

converges to 0. Using β_{20i} from (9) gives:

$$\beta_{20i} = \frac{1 - \frac{cov_0(z_i, p_i)}{rvar_0(p_i)}}{\rho(\sigma_i^2 - \frac{cov_0^2(z_i, p_i)}{var_0(p_i)})} = \frac{rvar_0(p_i) - cov_0(z_i, p_i)}{\rho(\sigma_i^2 var_0(p_i) - cov_0^2(z_i, p_i))},$$

is bounded from above, since the derivations in Appendix A.4 imply that $cov_0^2(z_i, p_i)$ is strictly smaller than $\sigma_i^2 var_0(p_i)$. This and boundedness of α_i immediately implies that

$$\lim_{\lambda_i \to \{0,1\}} \lambda_1 \beta_i = 0.$$

□

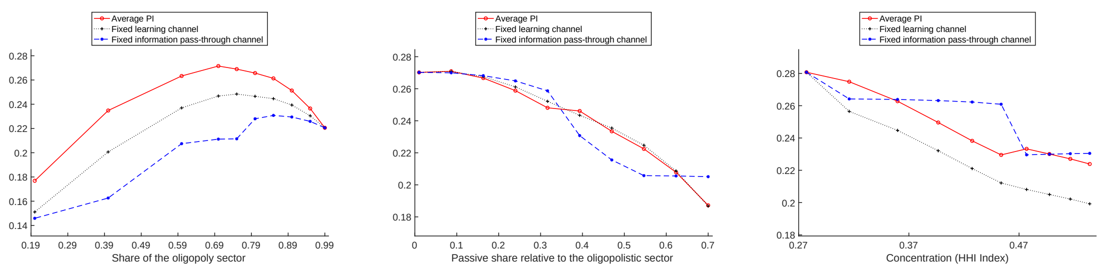
B Sensitivity and Robustness

B.1 DARA Version of the Model

In this section, we present the results of our three experiments in a version of the benchmark model in which risk aversion depends on investors' size. In particular, we posit that risk aversion of an oligopolist j is given by:

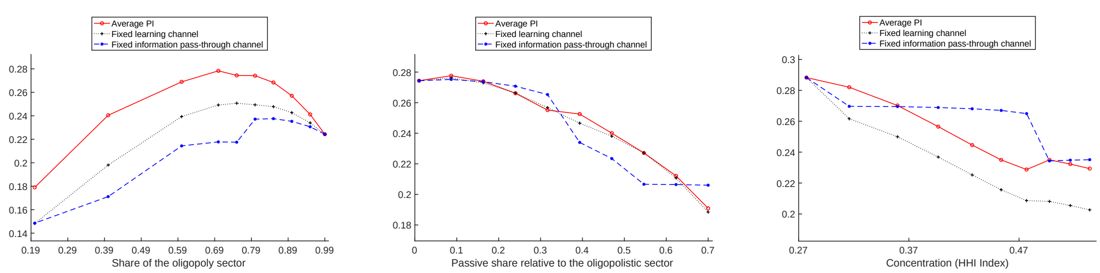
$$\rho_j = \bar{\rho} - s\lambda_j,$$

that is, it is decreasing in size. Apart from s, the parameterization of the model is identical to that of the benchmark, with $\bar{\rho} = \rho$ from Table 1. We present the results for two settings of the slope parameter, $s = 0.2$ and $s = 0.8$. The slope of 0.2 implies variation of risk aversion parameter ρ between the smallest and largest size of the oligopolist (across all parameterization) between baseline 2.32 and 2.21 for $s = 0.2$ and a much more substantial variation between 1.89 and 0.62 for $s = 0.8$.
Figures 18 and 19 present the results. Comparing these results with those in Figures 7, 9, and 11 in the benchmark model, we can see that the results and conclusions are qualitatively similar to the benchmark model. Decreasing the risk aversion with size does increase average holdings of oligopolists but does not shift the position of the size for which price informativeness is maximized. The small impact of introducing the DARA specification to the model can be understood intuitively by noticing that it is unlikely to affect the learning choices across assets, as it affects demand for all assets proportionately, and does not directly alter the relative benefit of learning about one asset versus another. As a result, introducing DARA does not significantly alter our results.



(a) (b) (c)

Figure 18: The figure presents the relationship between average price informativeness and oligopolistic sector size (panel (a)), the share of the passive sector (panel (b)), and size concentration (panel (c)). Slope parameter $s = 0.2$.



(a) (b) (c)

Figure 19: The figure presents the relationship between average price informativeness and oligopolistic sector size (panel (a)), the share of the passive sector (panel (b)), and size concentration (panel (c)). Slope parameter $s = 0.8$.

B.2 Version of the Model with Linear Capacity Constraint

In this section, we present a version of the model in which we replace the Shannon entropy capacity constraint, given by $\prod_{i=1}^{n} \alpha_{ji} \leq e^{2K_j}$, with a linear capacity constraint, similar in spirit to one used for example by Kacperczyk, Van Nieuwerburgh, and Veldkamp (2016):

$$\sum_{i=1}^{n} \alpha_{ji} \leq n + 2K_j,$$

where $\alpha_{ji} \geq 1$ and hence at $\sum_{i=1}^{n} \alpha_{ji} = n$, there is no learning. As for the parameterization, it differs from parameter choices in Table 1 only in the choice of capacity, which in this case needs to be recalibrated to match the average market return of between 6% and 7.5%, just like in the benchmark. This gives $K_j = 12.5, K_k = 0.125$ for $j \in LA, k \in SA$. The results from this version of the model are presented in Figure 20. The response of price informativeness for our three experiments is qualitatively consistent with the results from the benchmark model, although the magnitudes differ between the two models.

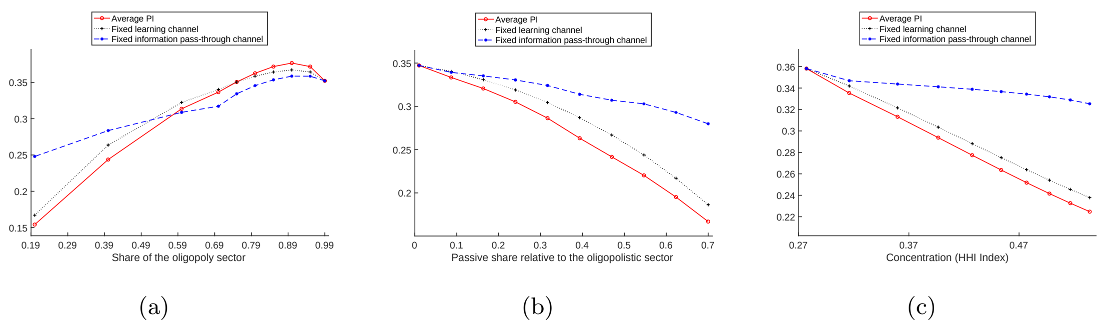


Figure 20: The figure presents the relationship between average price informativeness and oligopolistic sector size (panel (a)), the share of the passive sector (panel (b)), and size concentration (panel (c)), for linear entropy specification.

B.3 Extension to the Model with Endogenous Capacity Choice

Below, we present results from an extension of the model in which oligopolistic investors optimally choose their capacities, $K_j > 0$, subject to a convex cost. Specifically, we assume that small active oligopolists are followers and choose their capacity in response to the large oligopolist choices. To make the problem tractable, we restrict the choice of the small active oligopolist to be the same, and use oligopolist $j = 5$ as the decision maker. $\hat{K}_j$, $j \in SA$ is such that[46]

$$\hat{K}_j(K_1, K_2) = argmax_{K \in \mathcal{K}_{\text{small}}} E\lambda_5 U_5 - \phi K^a, \text{ given } K_1, K_2.$$

Large active oligopolists, that is, $j \in LA = \{1, 2\}$ internalize the optimal response of the small oligopolists, and choose a best response to each other's information capacity choice. That is,

$$\text{given } \hat{K}_2 \text{ and } \hat{K}_j(K_1, K_2), \hat{K}_1 = argmax_{K \in \mathcal{K}_{\text{large}}} E\lambda_1 U_1 - \phi K^a,$$

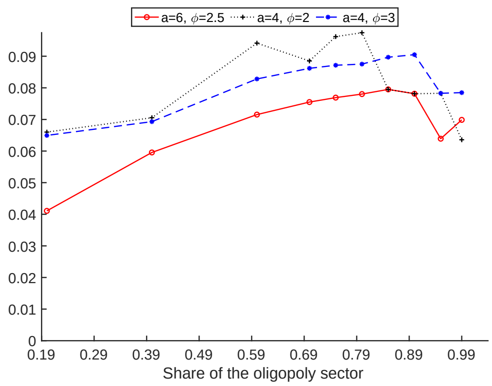
and

$$\text{given } \hat{K}_1 \text{ and } \hat{K}_j(K_1, K_2), \hat{K}_2 = argmax_{K \in \mathcal{K}_{\text{large}}} E\lambda_2 U_2 - \phi K^a.$$

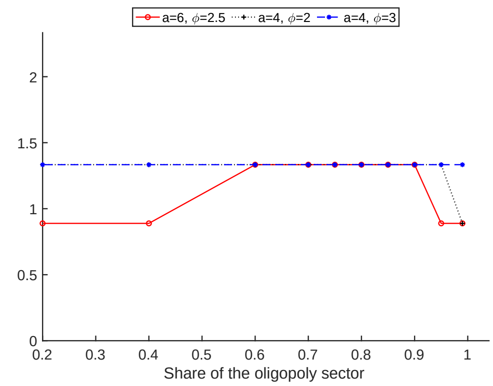
We discretize the K_{small} and K_{large} to be on a grid of size 10.[47]

[46] Due to similar sizes, the utilities of the small oligopolists are almost identical.
[47] Grid size is dictated by solution time, since even with size 10, the search involves solving the model 30000 times.

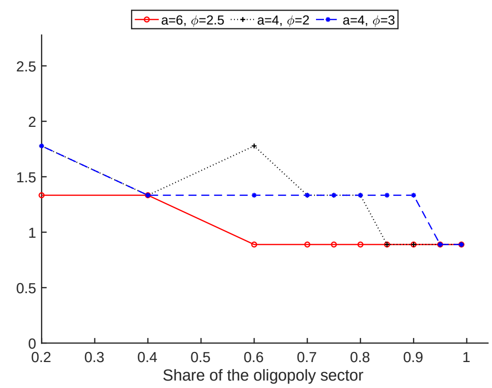
The results of the size experiment are presented in Figure 21. As the size of the sector grows, large oligopolists 1 and 2 choose a relatively stable information capacity for a variety of information cost curvature. Small oligopolists choose an increasing capacity, but much smaller in size compared to large oligopolists. Quantitatively, the optimal K choices do not move enough to overturn the general hump-shape of the average price informativeness curve. However, they imply that the peak of price informativeness moves to the right, as larger sizes imply larger capacity choices, which increases information production.



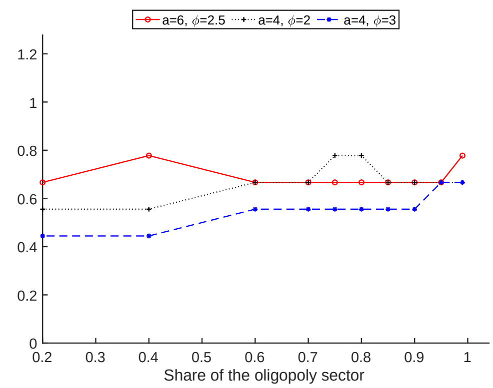
(a) Average price Informativeness



(b) Optimal K_1



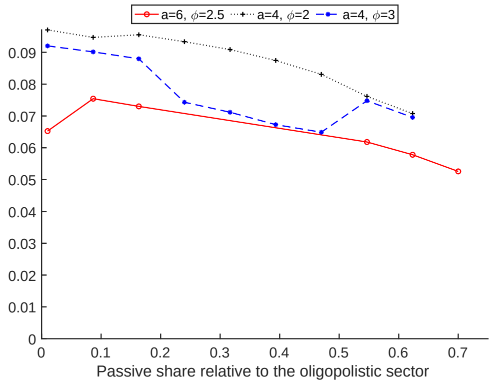
(c) Optimal K_2



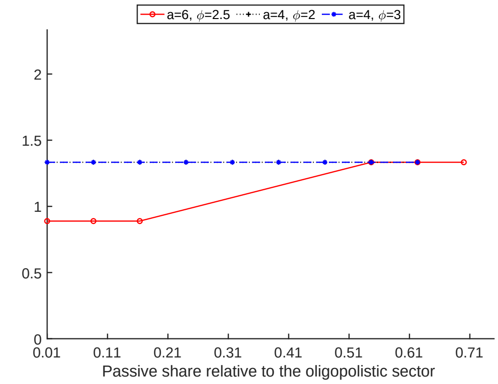
(d) Optimal K_j for small investors

Figure 21: Panel (a) shows the relationship between average price informativeness and the share of the oligopolistic sector, while panels (b) through (d) show the total chosen capacity for each large oligopolist and the small oligopolists relative to the share of the institutional investors. For each panel, the red and black lines differ in terms of curvature for the same scale parameter, and the blue line varies the scale parameter of the cost.

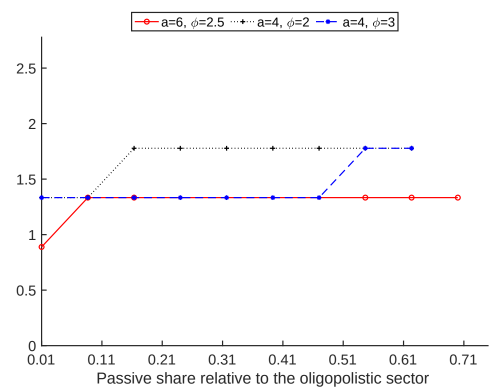
Figure 22 presents results for the growth of the passive sector. As the passive sector grows relative to active sector, large oligopolists keep their information choices stable, and the small active oligopolists choose to decrease their information capacities, reinforcing the decrease in the average price informativeness.
Finally, Figure 23 presents the results for the concentration case. As concentration increases, larger active oligopolist keeps their capacity choice stable or decreases it, while smaller (but still large) active oligopolists increases their choice. The small active oligopolists decrease their capacity choice due to their now smaller relative size. The net effect on price informativeness is an overall decreasing average price informativeness (panel a), but the magnitudes and the exact shape depend on the specification of the cost function.



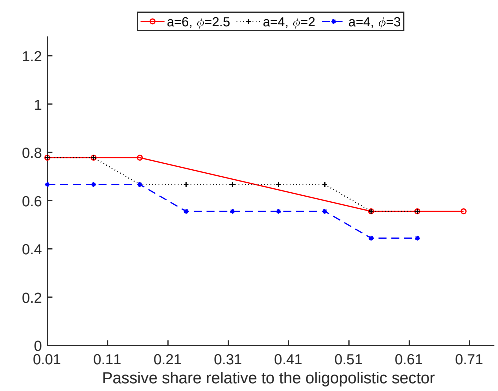
(a) Average price Informativeness



(b) Optimal K_1

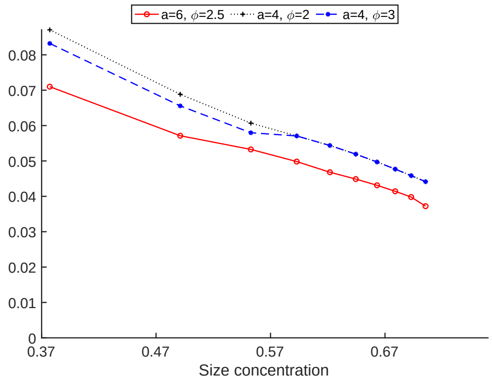


(c) Optimal K_2

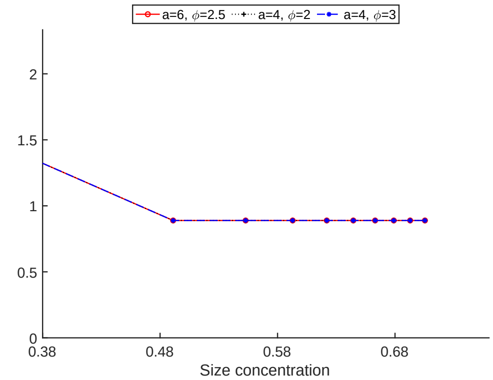


(d) Optimal K_j for small investors

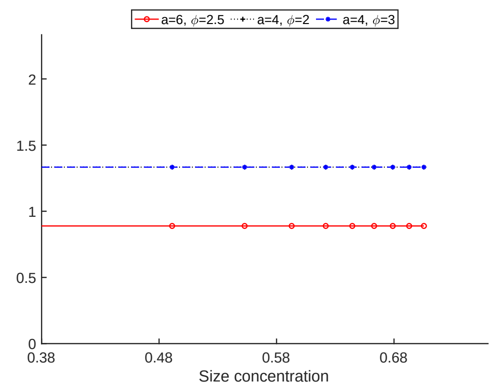
Figure 22: Panel (a) shows the relationship between average price informativeness and the share of the passive sector, while panels (b) through (d) show the total chosen capacity for each large oligopolist and the small oligopolists relative to the share of the passive sector. For each panel, the red and black lines differ in terms of curvature for the same scale parameter, and the blue line varies the scale parameter of the cost.



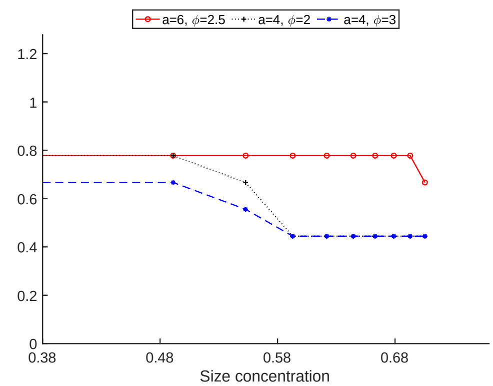
(a) Average price Informativeness



(b) Optimal K_1



(c) Optimal K_2



(d) Optimal K_j for small investors

Figure 23: Panel (a) shows the relationship between average price informativeness and the size concentration of the oligopolistic sector, while panels (b) through (d) show the total chosen capacity for each large oligopolist and small oligopolists relative to the concentration of the institutional sector. For each panel, the red and black lines differ in terms of curvature for the same scale parameter, and the blue line varies the scale parameter of the cost.

B.4 Sensitivity to Parameter Choices

Figures 24-27 present results from a set of experiments that vary our parameterization choices. In each instance, we reparameterize the risk aversion to match the market return. Each figure caption reports the relevant values. The rest of the parameters are as in the benchmark model.

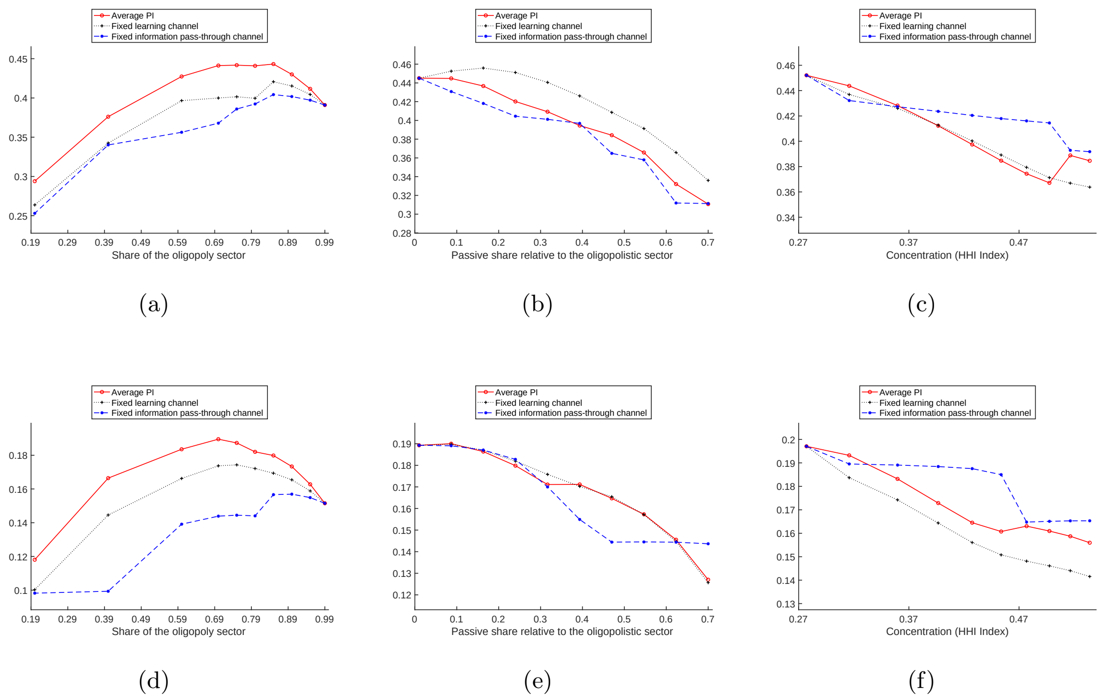


Figure 24: The figure presents the relationship between average price informativeness and oligopolistic sector size (panel (a) and (d)), the share of the passive sector (panel (b) and (e)), and size concentration (panel (c) and (f)), for $\bar{z} = 5, \rho = 1.13$ (panels (a)-(c) and $\bar{z} = 15, \rho = 3.5$ (panels ((d)-(f)) calibration. The rest of the parameters are as in the benchmark model.

B.5 Parameterization with Passive Indexers

Figure 28 presents the results from our three experiments for a parameterization of the model in which half of the passive investors are passive indexers, meaning that they hold the index, defined as an average supply-weighted portfolio, and are not price-sensitive. That essentially allocates a fraction of the overall supply of the asset, to the passive indexers, effectively changing the average supply of the assets faced by the remaining investors. In this experiment, we reparameterize the risk aversion to match the market return, which gives $\rho = 2.4$. The rest of the parameters are as in the benchmark model.

B.6 Model with Informed Retail Investors

In this section, we present results from a version of the model, in which we introduce small retail investors with positive capacity. Specifically, we parameterize the model to include 50 retail investors (on top of 20 oligopolists), with capacity equal to the capacity of the small active oligopolists. The retail investors are accounted as part of the non-institutional sector in the model, and hence introduce an additional effect of changing institutional sizes, namely that increased institutional ownership reduces ownership of informed retail investors, which can potentially impact the response of price informativeness.

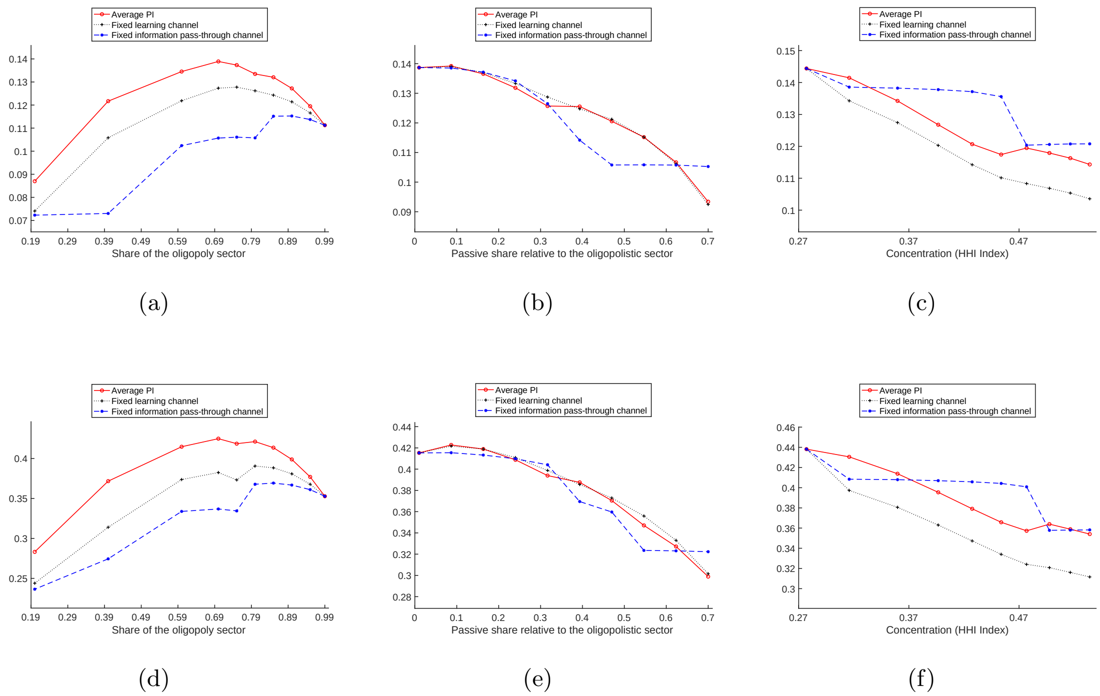


Figure 25: The figure presents the relationship between average price informativeness and oligopolistic sector size (panel (a) and (d)), the share of the passive sector (panel (b) and (e)), and size concentration (panel (c) and (f)), for $\sigma_i = 0.7\ \forall i$ and $\rho = 4.75$ (panels (a)-(c) and $\sigma_i = 1.3\ \forall i$ and $\rho = 1.37$ (panels ((d)-(f)) calibration. The rest of the parameters are as in the benchmark model.

The parameterization of sizes in this version of the model follows exactly the benchmark model for oligopolists. For retail investors and competitive uninformed fringe, we assume among the 50 retail investors, the largest is twice as large as the smallest. In terms of setting the size levels, we provide two parameterizations. In the first one, we assume that half of non-institutional ownership is in the hands of retail investors and the other half is in the hands of competitive fringe. In this case, we match market return by setting $\rho = 2.5$, and leave the other parameters as in benchmark. The results are presented in Figure 29. In the second experiment, we set the retail sector size so that they constitute 98% of non-institutional size. As before, we reparameterize $\rho = 2.63$ and leave the remaining parameters as in the benchmark model. Figure 30 present the results.

The predictions of the model with informed retail investors differ quantitatively from the benchmark model but are close. Intuitively, this is due to several factors. First, the size of each retail investor is small. That means that they won't have a high information pass-through due to their small economic significance. Also, small size means they will endogenously specialize in learning about only one asset, which further diminishes their impact on price informativeness. Finally, shifting ownership from small oligopolists to retail should in principle have very small effect on price informativeness, and so the only difference comes from shrinking large active oligopolists at the benefit of retail investors.

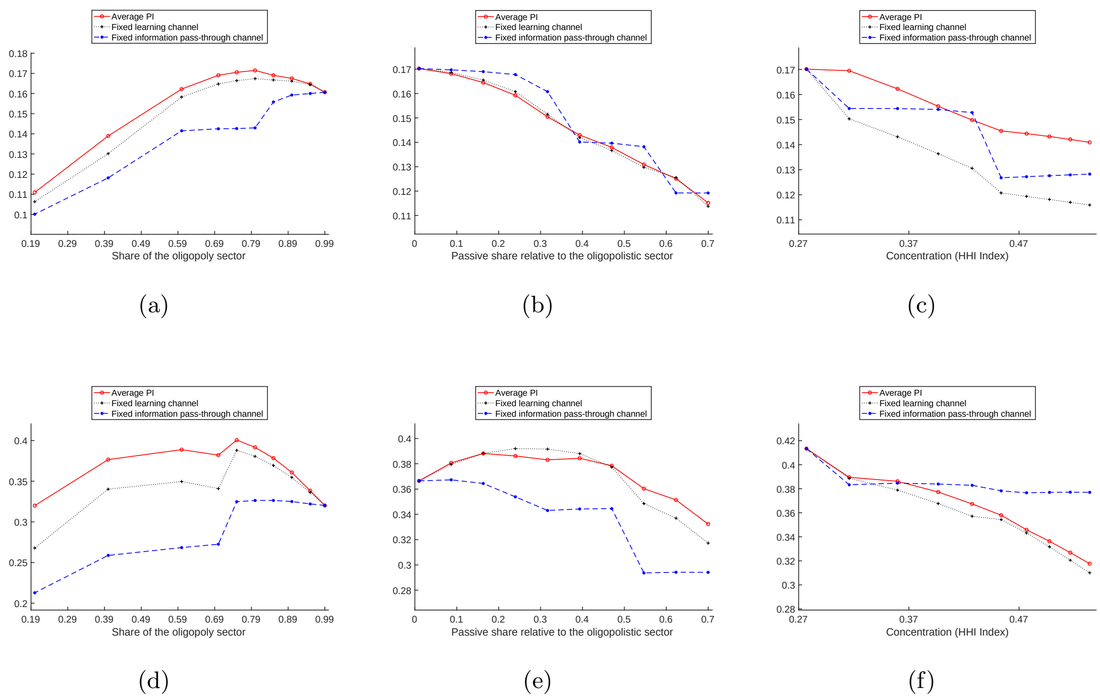


Figure 26: The figure presents the relationship between average price informativeness and oligopolistic sector size (panel (a) and (d)), the share of the passive sector (panel (b) and (e)), and size concentration (panel (c) and (f)), for $K_j = 2, K_k = 0.2$, $j \in LA, k \in SA$ and $\rho = 1.7$ (panels (a)-(c) and $K_j = 6, K_k = 0.6$, $j \in LA, k \in SA$ and $\rho = 3.05$ (panels ((d)-(f)) calibration. The rest of the parameters are as in the benchmark model.

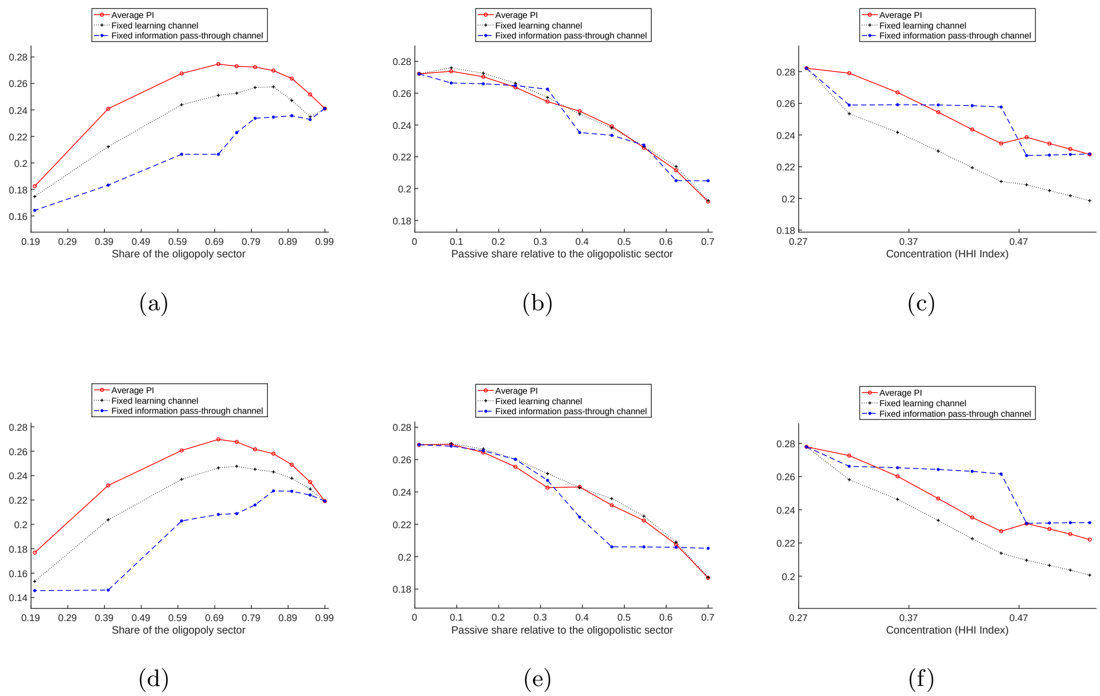


Figure 27: The figure presents the relationship between average price informativeness and oligopolistic sector size (panel (a) and (d)), the share of the passive sector (panel (b) and (e)), and size concentration (panel (c) and (f)), for $\bar{x}_i \in [1, 4]$ and $\rho = 4.3$ (panels (a)-(c) and $\bar{x}_i = 12$ and $\rho = 1.56$ (panels ((d)-(f)) calibration. The rest of the parameters are as in the benchmark model.

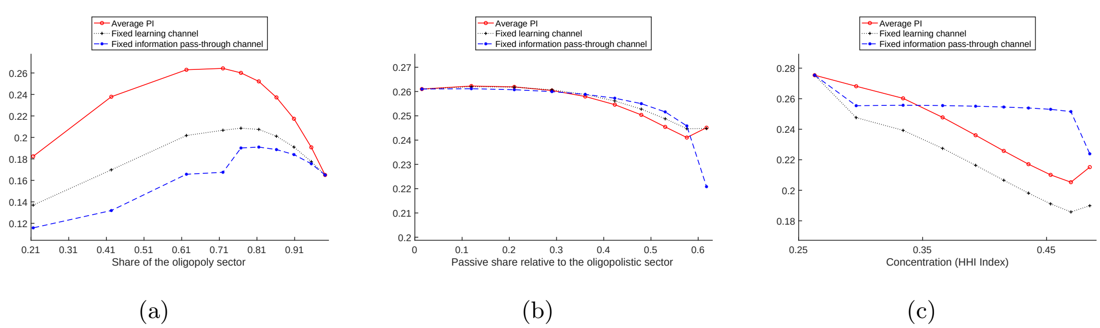
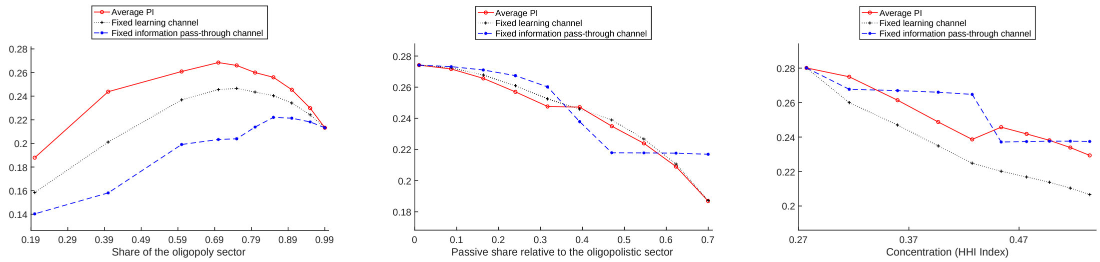
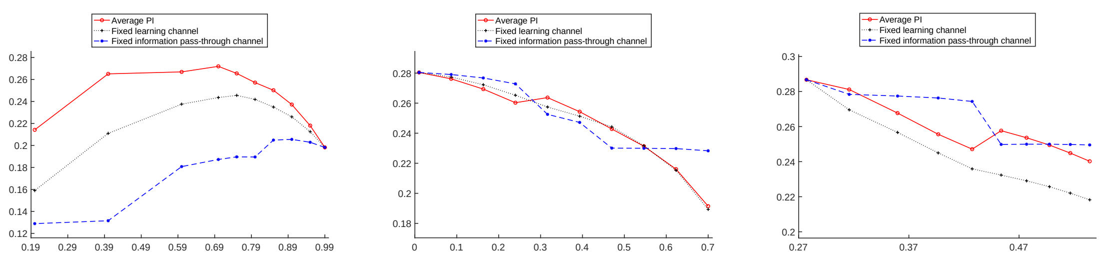


Figure 28: The figure presents the relationship between average price informativeness and oligopolistic sector size (panel (a)), the share of the passive sector (panel (b)), and size concentration (panel (c)) for the version of the model with passive indexers.



(a) (b) (c)

Figure 29: The figure presents the relationship between average price informativeness and oligopolistic sector size (panel (a)), the share of the passive sector (panel (b)), and size concentration (panel (c)) for the version of the model with retail informed investors.



(a) (b) (c)

Figure 30: The figure presents the relationship between average price informativeness and oligopolistic sector size (panel (a)), the share of the passive sector (panel (b)), and size concentration (panel (c)) for the version of the model with retail informed investors.

B.7 Additional Figures

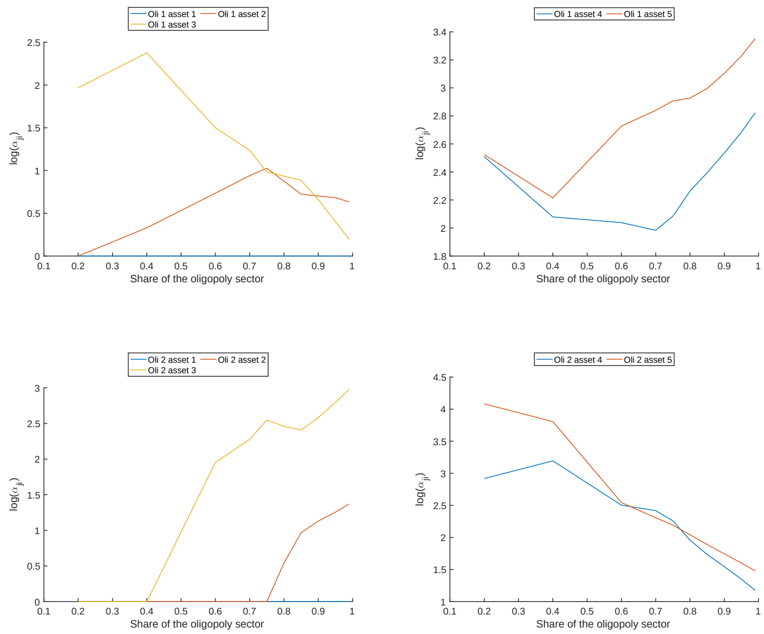


Figure 31: The figure presents the relationship between information choices and oligopolistic sector size for two largest oligopolists in the benchmark model.

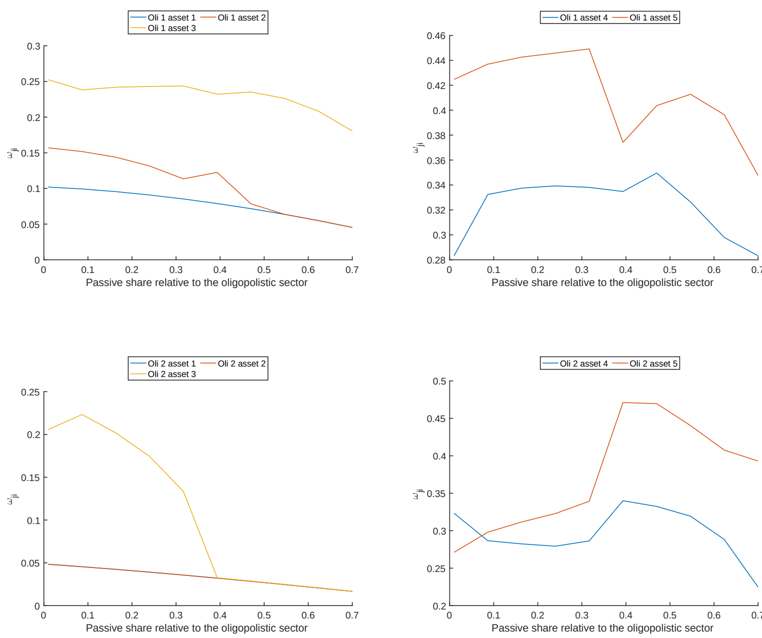


Figure 32: The figure presents the relationship between information pass-through and passive share two largest oligopolists in the benchmark model.

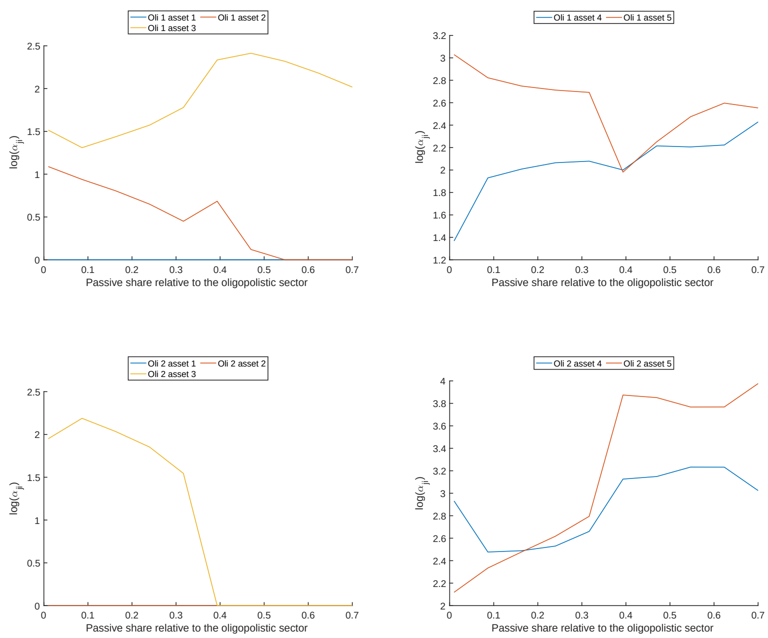
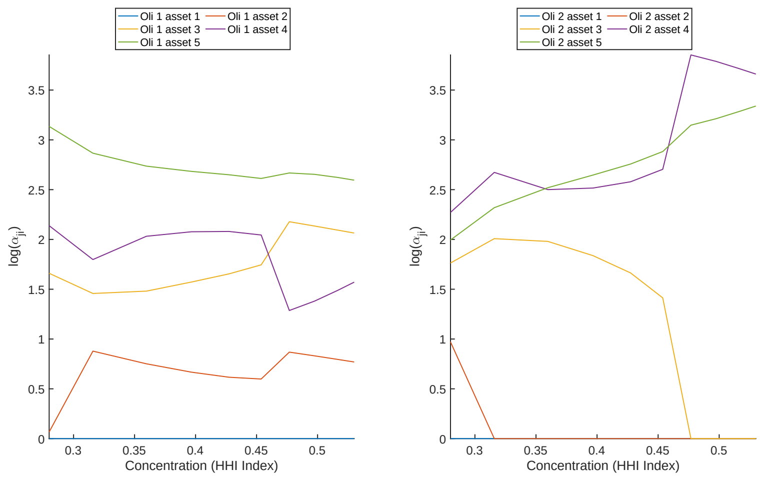
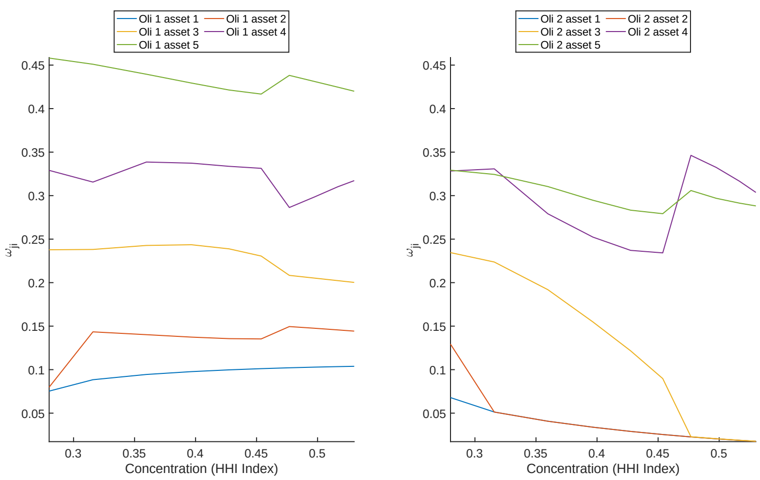


Figure 33: The figure presents the relationship between information choices and passive share two largest oligopolists in the benchmark model.



(a)



(b)

Figure 34: The figure presents the relationship between information choices (panel (a)), information pass-through (panel (b)) of the two largest oligopolists and concentration in the benchmark model.

C Computational Algorithm

C.1 Solving for βs, given information allocation

For a given information allocation of the active oligopolists $\alpha_{ji}, i = 1, ..., n, j \in SA \cup LA$

1. Guess β_{1ji}, β_{2ji} for $i = 1, ..., n, j = 1, ..., l$.
2. Compute new β_{1ji}, β_{2ji} for $i = 1, ..., n, j = 1, ..., l$ using equations (8) and (9). We do not need to know β_{0ji}s for this step.
3. Update guess for β_{1ji}, β_{2ji} for $i = 1, ..., n, j = 1, ..., l$.
4. Repeat steps 2-3 until convergence.
5. Compute analytically β_{0ji} for $i = 1, ..., n, j = 1, ..., l$ using (7).

C.2 Solving for information allocation

1. Take a guess $\alpha_{ji}, i = 1, ..., n, j \in SA \cup LA$ ($\alpha_{ji} \equiv 1, i = 1, ..., n, j \in SP \cup LP$).
2. For oligopolist $k \in SA \cup LA$
   (a) Take $\alpha_{ji}, i = 1, ..., n$, and $j \neq k$ as given.
   (b) Solve for $\alpha_{ki}, i = 1, ..., n$ by maximizing (34) subject to capacity constraint $\prod_{i=1}^{n} \alpha_{ki} \leq e^{2K_k}$ and subject to betas endogenously responding to changing information via fixed point in C.1.
   (c) Update guess for $\alpha_{ki}, i = 1, ..., n$.
3. Repeat step 2 for all $k \in SA \cup LA$, which gives new $\alpha_{ji}, i = 1, ..., n, j \in SA \cup LA$.
4. Repeat steps 2-3 until convergence.